Psychological Methods

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# Utilizing Topology to Generate and Test Theories of Change

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Statistical and methodological innovations in the study of change are advancing rapidly, and visual tools have become an important component in model building and testing. Graphical representations such as path diagrams are necessary, but may be insufficient in the case of complex theories and models. Topology is a visual tool that connects theory and testable equations believed to capture the theorized patterns of change. Although some prior work has made use of topologies, these representations have often been generated as a result of the tested models. This article argues that utilizing topology a priori, when developing a theory, and applying analogous statistical models is a prudent method to conduct research. This article reviews topology by demonstrating how to build a topological representation of a theory and recover the implied equations, ultimately facilitating the transition from complex theory to testable model. Finally, topologies can guide researchers as they adjust or expand their theories in light of recent model testing.

*Keywords:* dynamical systems, modeling, topology, change



Psychological scientists develop formal descriptions of human behavior and psychological processes by generating theories, translating theories into statistical models, and testing these models against empirical data. This process has proved challenging because our theories often depict complex nonlinear patterns of change within and between individuals over time, but our statistical techniques only capture a small portion of our theories. Sewall Wright's (1921) innovation of path diagrams provided a method of using visual tools to generate model and equation forms that fundamentally changed psychological research by equipping researchers with a method to express their theory in flow chart form and test their model against data. Techniques such as scatterplots in regression further anchored our understanding of the math that links theory to data (Friendly & Denis, 2005). As a result of these advances, statistical models were able to capture multiple simultaneous relationships, but struggle with complex nonlinear patterns and changes within and between people over time.

In recent years, researchers developed models such as dynamic factor analysis (DFA; Molenaar, 1985; Molenaar & Campbell, 2009; Molenaar, De Gooijer, & Schmitz, 1992; Wood & Brown, 1994), latent difference score modeling (LDS; Hamagami & McArdle, 2001; Hawley, Ringo Ho, Zuroff, & Blatt, 2006; King et al., 2006; McArdle, 2001, 2009), and latent differential equation modeling (LDE; Boker, Deboeck, Edler, & Keel, 2010; Boker, Neale, & Rausch, 2003; Butner & Story, 2011; Chow, Ram, Boker, Fujita, & Clore, 2005; Deboeck, Montpetit, Bergeman, & Boker, 2009; Helm, Sbarra, & Ferrer, 2012; Nicholson, Deboeck, Farris, Boker, & Borkowski, 2011) to capture nonlinear patterns and changes within and between people over time. Unfortunately, these models have advanced to such a degree that our current graphical tools may be insufficient to depict them. Whereas these advanced models can represent complex nonlinear theories, the path diagrams they generate are virtually incomprehensible, as they require complicated model structures to represent change (as is the case for LDS) or data transformations that obscure interpretation (the case for LDE and DFA). As a result, researchers may find it difficult to translate their theories into these advanced statistical models.

In this article we propose the use of an additional graphical representation that is capable of capturing complex theories and seamlessly translating them into testable models. This additional graphical representation is called a *topology*, and is typically communicated as an elevation map of some geographical terrain. More formally, a topology is a graphical representation of differential equations, sometimes called a *state space*, *phase space*, *vector field*, or *phase portrait* (Abraham & Shaw, 1992; Baker & Gollub, 1996; Gottman, Murray, Swanson, Tyson, & Swanson, 2002; Kantz & Schreiber, 1997; Kelso, 1995; Kugler & Turvey, 1987). Therefore, topology is not confined to merely describing one's change in elevation as one traverse's some terrain, but can be applied more generally to describe anything, like behavior or psychological constructs, that changes over time.

We will argue that a topological map of one's theory can aid in translating the theory into a testable statistical model. First, topologies can be used to represent very complex and nonlinear theories—like many theories of psychological processes. Second, topological maps are generated with mathematical equations (much like fitted lines in scatterplots), from which a path diagram and formal equation can be derived. Therefore topological maps can

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bridge the gap between complex theory and often intractable statistical techniques.

The connection between topological maps and statistical models of change has been made by others (Boker & Nesselroade, 2002; Gottman et al., 2002; Liebovitch, Peluso, Norman, Su, & Gottman, 2011). Researchers who use LDS, LDE, and DFA techniques often describe the topological features implied by a model, but the order of progress presented in articles is often theory, statistical model, and then implied topology. Generating topologies as a first step in model building will help bridge the gap between theory and statistical models. Researchers less familiar with topologies and statistics may assume that an equation needs to be established before graphically displaying it as a topological map. Instead, we suggest that the order should be reversed in the scientific process. By reversing the order, researchers would theorize using a topological map and then derive the statistical equation to test the full extent of their theory—similar to the way path diagrams are used in conjunction with structural equation modeling today. In doing so, researchers may find that topology closes the gap between theory and statistical model without conceding the richness of their theory.

The goal of this article is to provide a tutorial on how to move from theory to topology, and from topology to testable equation form. We will integrate our approach with recent advanced statistical models and demonstrate how these models can be simplified via topology, making them more accessible. In order to tie our proposed process together, we will use a continuing example with real data that investigates the complex relationship of dyadic affect regulation. This example is meant to highlight major steps in the process of moving from theory to topology and ultimately to testable equation. We also provide a glossary of terms in the Appendix, since much of the jargon from topology may be unfamiliar to the reader.

## A Case for Topology

Psychological researchers are faced with the daunting task of explaining and predicting behavior given a potentially infinite number of variables (Meehl, 1978). Cronbach (1975) summarized the sheer complexity of trying to capture human behavior by stating: “Once we attend to interactions, we enter a hall of mirrors that extends to infinity” (p. 119). In other words, there are nearly an infinite number of possible interactions, and many of the phenomena observed in the laboratory may be very different if these higher order interactions are taken into account. Cronbach urged scientists to be explicit about the precise context under which a variable is being observed, as it will likely change in the presence of other unobserved variables. As a result, many large effect sizes established in laboratory settings were absent once the construct of interest was tested outside the restricted context in which it was observed (Shoda & Mischel, 2000). Cronbach posited that in the real world, the problem of identifying a causal relationship may be intractable given the potential of many variables interacting within and between individuals over time.

One conclusion from Cronbach’s critique is dismay; another and more productive outcome is a reexamination of our methods and statistics that may be unknowingly limiting tests of our theory (Watson, 1913). For generations psychology has relied heavily on statistical models such as analysis of variance and other varieties of the general linear model, which inherently constrains our ability to assess the fit of our complex theory to the empirical data. Models of change, on the other hand, may potentially resolve Cronbach’s hall of mirrors or at least resolve some of the complexity.

LDS, LDE, and DFA are a series of statistical models that focus on change in constructs over time. Many of these models are derived from dynamical systems theory, a theory on how multicomponent systems interact to form emergent complex patterns of change through time. In a systems approach, the higher order interactions referred to by Cronbach constitute emergent patterning where multiple variables and contexts are pushing and pulling one another in a coordinated dance over time. This dance generates temporal patterning that is descriptive of the overall system, and these patterns can be depicted through models of change.

In sum, psychological processes are often more complex than the statistical models used to represent them. Contemporary models have been developed that are capable of reflecting more complex and nonlinear theories (Boker, 2001; Boker & Nesselroade, 2002; Butner, Amazeen, & Mulvey, 2005; McArdle, 2001), and idiographic approaches have also been developed that allow for the analysis of intraindividual change (Hamaker, Dolan, & Molenaar, 2005; Molenaar, 2004; Molenaar & Campbell, 2009; Nesselroade & Molenaar, 2010). These advanced models of change may resolve these issues, whether through autoregressive, change score, differential equation estimation, or direct derivative estimation approaches. Despite the fact that these techniques are beginning to capture the complexity that our theories demand, there are still challenges in translating our theories into testable statistical models without extensive statistical training.

To solve these problems, we propose using a topological map as a midstep between theory and testable equation. As a graphical representation of change, topological maps can capture all of the techniques (e.g., LDE, DFA) developed to deal with nonlinear and intraindividual changes and can therefore be used to represent complex theories. In addition, all changes over time, within and between people, are captured by the same topological representation. Finally, after testing one’s theory via topology, expansions and adjustments to the theory can be aided by the topological representation. In the next section we introduce the idea of a topology and demonstrate how to translate theory into a topological representation.

## Topology

A topology is a graphical representation of differential equations, sometimes called a state space, phase space, vector field, or phase portrait (Abraham & Shaw, 1992). These tools are often central to dynamical systems theory, but more importantly many of the advanced methods, such as LDS and LDE, imply a topology. In this tutorial we will focus on building a topology that reflects a theory and then examine the equivalent equation form. Although topologies have precise mathematical definitions, we will begin our discussion with the more colloquial understanding of a topology. Figure 1 is a topographical contour map of the area behind the University of Utah. The lines help identify altitude so that we can get a sense of mountains and valleys—they show a third dimension in a two-dimensional representation (north–south being the *y*-axis

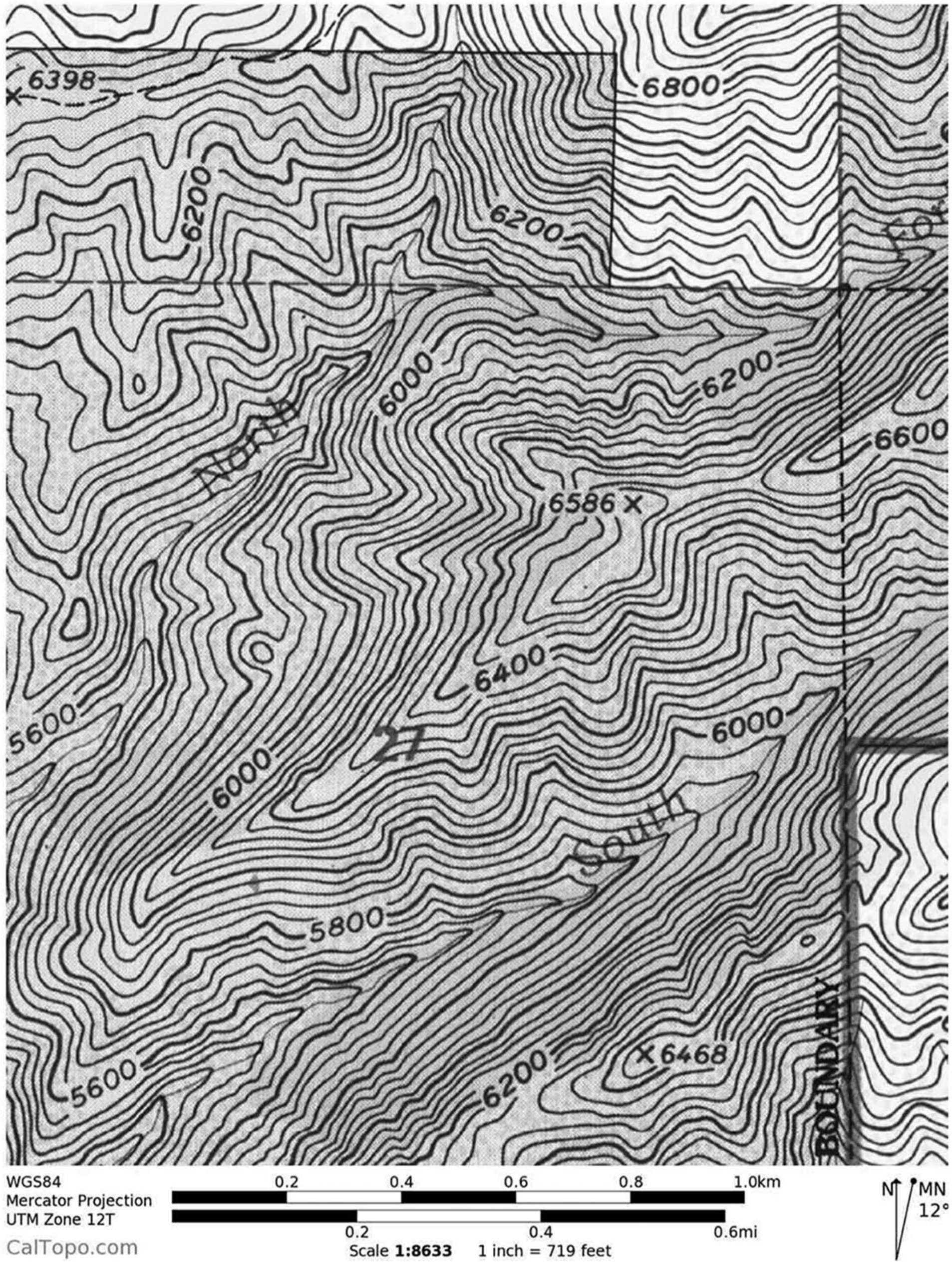



*Figure 1.* A topographical map of an area behind the University of Utah (U.S. Geological Survey 7.5′ map generated with CalTopo.com; http://caltopo.com/map.html#ll=40.79014,-111.82072&z=16&b=t) depicting mountains and ridges with valleys between them.

and east–west being the *x*-axis, where the contour lines represent the third dimension of altitude).

If we were to imagine hiking in this area, we can see that valleys, peaks, and ridges would impact the trajectory of our hike in terms of the probabilities and ease of where we are likely to wander. The mountains and ridges, in essence, guide the likely paths we would follow. It is important to realize that they do not entirely constrain where we might go, but rather capture a degree of likelihood.

During this hike we wear a Global Positioning System that tracks our location every minute, and from this it is possible to create vectors showing where we are, where we are going, and how fast we are moving at any point in time. Arrows going downhill are going to be longer because we would be walking faster than on flat terrain, where the arrows would be shorter. In our trek we might also walk uphill, creating substantially shorter vectors because of our slowed pace. Now imagine repeating this process until we have started our trek at all possible places on the map, going in all possible directions from those places. We would then have a map covered in vectors that describe where we are likely to go over time given a particular starting location. The vector field might look something like Figure 2. Notice that only the longest vectors from each location are represented on the map. In our example, we can imagine that our path led us both up and down the same hill, with the uphill vectors being much shorter than the downhill vectors. In this instance only the downhill vectors are displayed because the downhill vectors also represent what we would have to overcome in order to walk uphill. Therefore, the vectors displayed represent a combination of the likely path we would follow from the area local to a given vector and the effort it would take to diverge from this path.[1]

## A One-Dimensional Topology

Applying topology to psychology merely requires replacing direction (north/south and west/east) with psychological variables of interest. To understand all the underlying concepts, we will focus on a single-outcome, negative affect from a study of 48 happy couples who completed a daily diary survey for 21 consecutive days (this sample is reported in detail in Butner, Diamond, & Hicks, 2007). Affect and emotion regulation can both impair and enhance mental and physical health (see Diamond & Hicks, 2004, for a review), and thus how individuals are able to manage negative affectivity can have far-reaching implications. From an individual viewpoint, part of this is a process of staving off and buffering events that lead to negative affectivity so that their impact is limited in scope and time. Interpersonal interactions, especially from romantic partners, are believed to be central to buffering and also sometimes incite these negative affectivity occurrences (Gable, Reis, & Downey, 2003; Reis, Sheldon, Gable, Roscoe, & Ryan, 2000). We will begin with a relatively simplistic representation of negative affect and build up to a complex theory of how couples might interact graphically and test the resultant equations to the data.

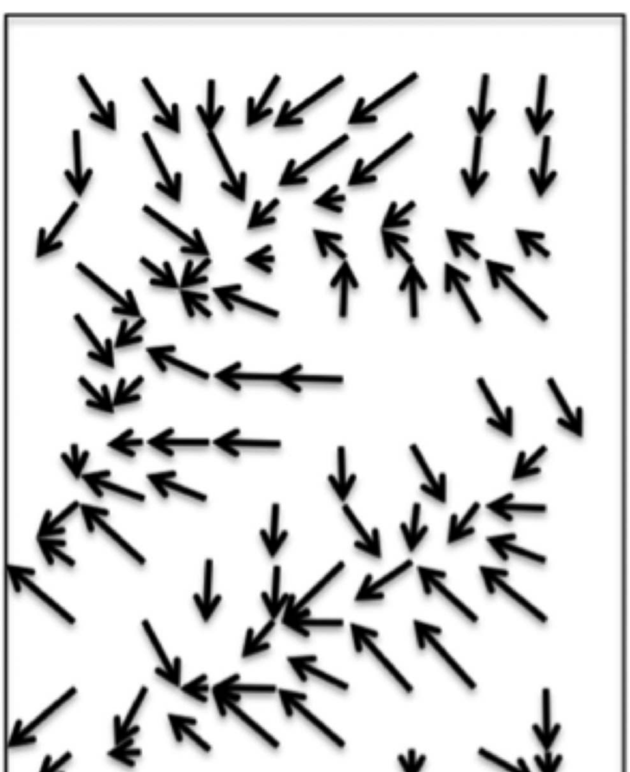


*Figure 2.* A velocity flow field of Figure 1 where the arrows show the likely progression and relative rate at which our path would change from a grid of starting locales of Figure 1.

Figure 3 shows a hypothetical graphical representation of where negative affect tends to move over time. Graphically representing the possible values as well as the change in the values of a variable is called a state space, phase space, or vector field (the phase portrait is the integral of the vector field and thus is read in the same way as the topographical map of Utah). Notice that the plot is rather difficult to read because the vectors are occluding the scale line for negative affect. To help us read the graph, we placed the hidden dimension of change (i.e., the vector lengths) on the *y*-axis where positive change is represented by higher values of change and negative change is represented by lower values of change in Figure 4.

The two-dimensional representation of a one-dimensional state space shows several key concepts. In Figure 4, we included a horizontal line where no change would occur. This is analogous to the lowest point in a one-dimensional valley. In theory all values of negative affect would move toward this value of negative affect through time—the homeostatic target usually a report of low negative affect.[2] These are points of no change, valleys, sinks, or attractors (Abraham & Shaw, 1992; Baker & Gollub, 1996; Kugler & Turvey, 1987), because the variable (e.g., negative affect) moves toward this point in time. More generically, this point is called a *set point* because all the behavior of the system (e.g., negative affect) is depicted in relation to this point.

Attractors are not the only topological feature in a one-dimensional state space. A value of negative affect that we rarely observe and are driven away from is known as a repeller. Like attractors, repellers also have a set point, which is like a mountain peak. We could stand at the peak of the mountain, but the moment we begin to move in any direction, we quickly move away from the peak. So, repellers generate change away from the set point rather than toward it. In a one-dimensional topology repellers function as the borders between two attractors. Therefore, regions on one side of the repeller will share the same movement through time (e.g., toward the same attractor) and those on the other side of the repeller will share a different region of collective movement through time. In this case, Figure 3 would show arrows moving away from the set point.

Figure 4 also shows how topology is directly related to models of change, since the vectors of where and how fast the value of

[1] This map would capture the behavior of many folks walking through this area under many circumstances. However, we can actually imagine a different vector map from a rock climber with bad knees, for example, who might hike much faster up the mountains and be cautious on the declines. That is, we can imagine variables that can alter the features of the map itself.

[2] The proper representation for a homeostatic target of negative affect would likely be no negative affect. However, we wanted to show how values would be pulled from both sides (values above and below) toward the target over time.

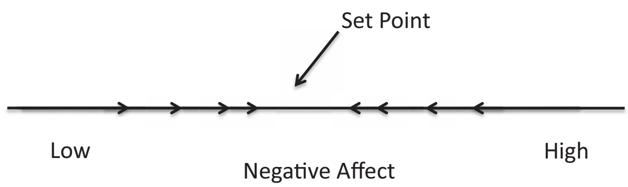



*Figure 3.* A one-dimensional state space where arrows point to where values of negative affect move toward in time.

negative affect changes in time are captured through the hypothetical hidden dimensions, much like altitude. Unlike a hypothetical time series we might observe, time is implicit in topology in that we can start at any point and ask where one would likely end up. Furthermore, we can begin to see the relationship between data, analysis, and topology in that Figure 4 could be implied from a simple scatterplot relationship where change in negative affect is the outcome and current negative affect is the predictor. Figure 5 shows a time series and a scatterplot of one participant's negative affect over time, and the subsequent best fitting straight line. Notice that in the time series data (Figure 5A), negative affect tends to move toward lower values (e.g., ~1.25), and these are the same values where we expect no change (e.g., an attractor) on Figure 5B. In the Equations of Change section, we will explicitly detail the translations between topology and equation.

The one-dimensional representation of negative affect is overly simplistic for our theory of affect regulation in that it depicts individuals as being drawn toward the same level of negative affect—the same set point or even a single individual homeostatically hovering around a single value. It may not be theoretically appropriate to assume that all individuals will settle on the same level of negative affect. Nor may it be appropriate to assume that a single individual will function homeostatically—it might be difficult to calm down once negative affectivity begins. This requires a topology that allows for different attractive homeostatic values of negative affect (i.e., different set points). Figure 6 is an example of a one-dimensional state space, a two-dimensional analog showing the hidden dimension of change, and an actual individual's data who showed this kind of behavior in his or her daily diary. Here we can envision observing a time series stuck in one basin of attraction (i.e., low negative affect) or the other basin of attraction (i.e., high negative affect) while also representing the difficultly of moving between them. In order for the value of negative affect for an individual to move from one attractor to another requires that it overcome the repulsive force of the repeller that exists between the attractors. In essence, the repeller depicts the resistance to calm down or get stuck in a state of negativity.

Behaviorally, the topology implies trajectories that hover around low negative affect, hover around high negative affect, and switch between them (albeit requiring the ability to overcome the repeller). Allowing for multiple set points demonstrates how a topology can allow for different people or the same person at different points in time (as is illustrated in the person's data in Figure 6) to exhibit different behaviors within the same topology, and ultimately the same equation.

Containing multiple set points is not the only topological feature that can be adjusted to meet the demands of a complex theory, but the strength and weakness of set points can be altered as well. Consider Vallacher, Nowak, Froehlich, and Rockloff's (2002) examination of the strength of positive self-evaluations as a function of positive ideations. In this study, participants moved a mouse around a central target on a monitor while listening to a prior recording of themselves. The prior recording consisted of thoughts the participants had expressed about themselves. Participants were instructed to move the mouse closer to the central target when they felt that a thought they heard in their recording was positive and away from the central target when they felt a thought was negative. Prior to listening to the recordings, participants were primed to recall either positive or negative past actions (or not primed to recall past actions). Participants who were primed to recall positive past actions moved the mouse closer to the central target more often than the negatively primed participants. In doing so, the time series recording of the mouse position for those in the positive prime condition created longer vectors that were directed toward the central target. This demonstrates that a set point can be strengthened or weakened based on an additional variable (i.e., positive/negative prime).

Variables that have the capacity to alter the topology are known as control parameters (Abraham & Shaw, 1992; Butner & Story, 2011; Kugler & Turvey, 1987). Control parameters have the ability to alter topological features in one of three ways. First, they can strengthen or weaken an attractor or repeller. Second, they can move a set point to a different location relative to other set points. Third, control parameters can drastically change the topology by completely extinguishing set points or turning it into a different kind of topological feature (e.g., change an attractor into a repeller, or vice versa). The flexibility to allow additional variables to alter the topology makes this graphical representation ideal for capturing the more intricate features of our theories.

Knowing that topological features can be altered in a variety of ways, let us return to Figure 5 depicting a single attractor for negative affect. In Figure 5, we plotted the change in negative affect at time (t) on the *y*-axis and negative affect at time (t) on the *x*-axis. Where the line crossed the 0 value on the *y*-axis was the point at which negative affect ceased to change, meaning it was the set point. Theoretically, we can imagine that the strength of one's set point for negative affect might be altered by their partner's negative affect, capturing how partners are able to buffer or exacerbate problems (Campbell, Simpson, Boldry, & Kashy, 2005; Collins & Feeney, 2000).

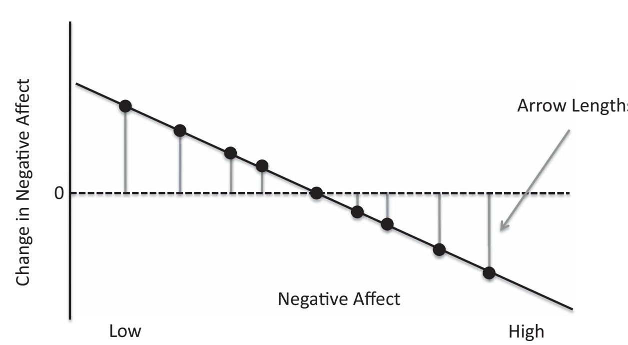



*Figure 4.* A one-dimensional state space where the hidden dimension of change in negative affect is shown on the *y*-axis. The occluded horizontal arrows from Figure 3 correspond to the vertical lines in Figure 4. In this case, all changes fall on a straight line.

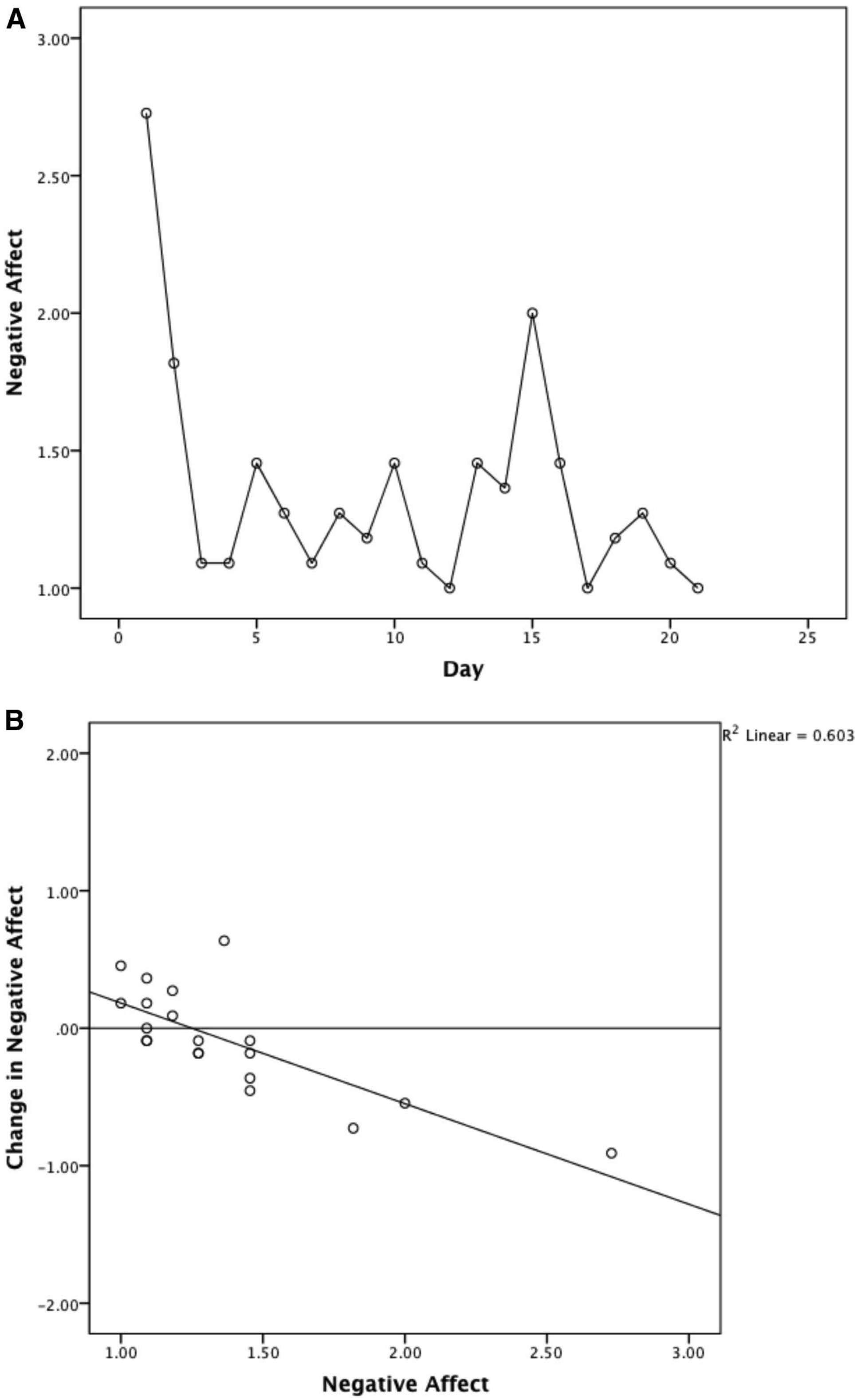



*Figure 5.* A time series (A) and scatterplot (B) representation of one person's negative affect over 21 days. In the state space (B), where the regression line crosses the 0 value (0 change) is the set point. The slope of the line indicates attractiveness/repulsiveness and strength of attraction. In the time series, the set point is the value of negative affect that the data keep returning to in time. Extreme deviations exist but are followed by a change back toward that point.

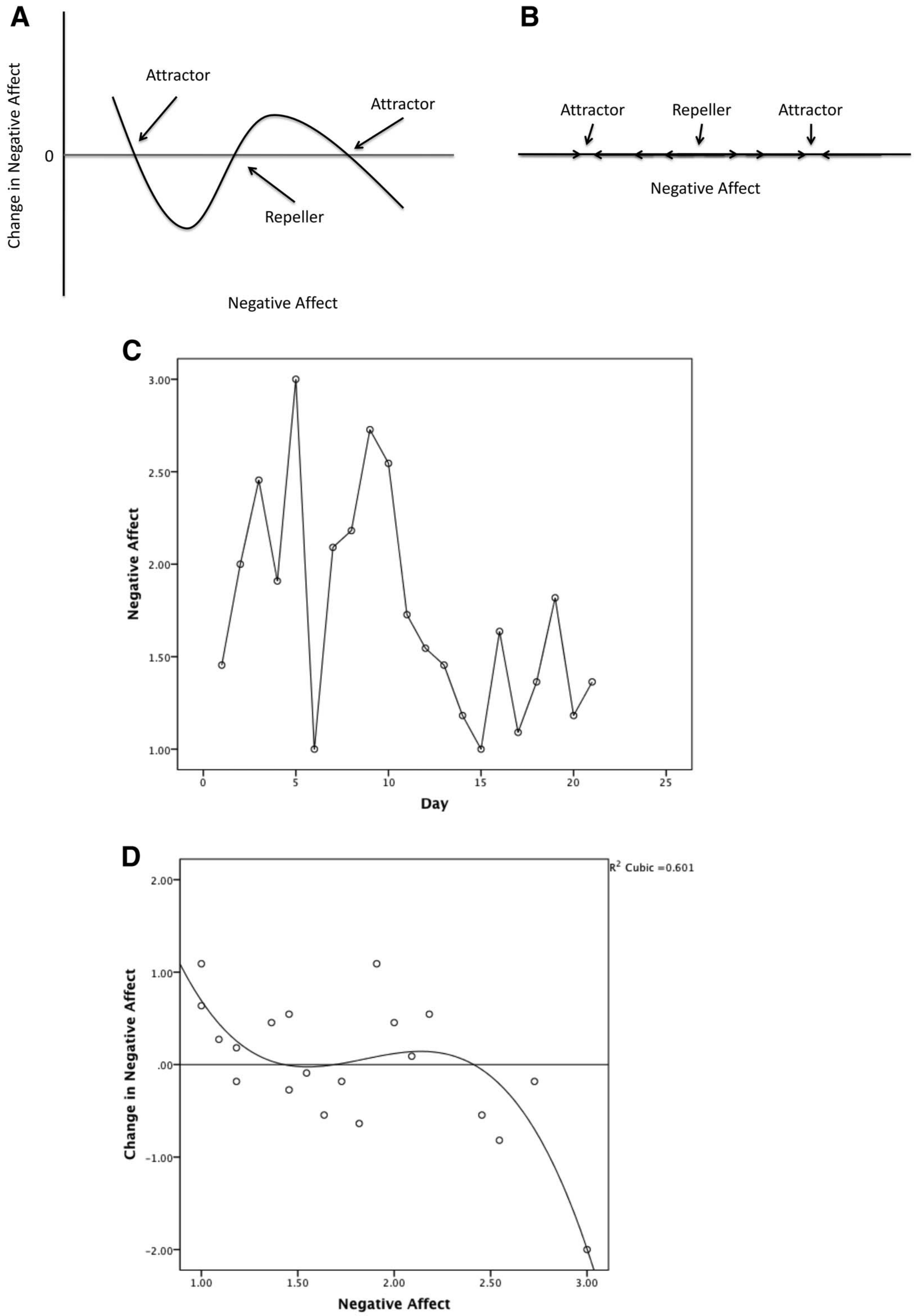



*Figure 6.* A one-dimensional state space with (A) and without (B) the hidden dimension of change shown with two stable attractors at high and low negative affect and a repeller separating them. The figure shows a time series of one individual (C) and the state space of that same individual (D) showing two attractors—one at low negative affect and one at moderate negative affect. This individual showed a pattern of getting stuck at moderate negative affect sometimes with the repeller indicating the resistance of switching between the states.

Figure 7 depicts the change in negative affect for females as a function of their male partner's level of negative affect (this graph was generated from the results of a multilevel model with female's affect, male's affect, and a female by male affect interaction predicting female's change in negative affect). The line with the steeper slope (partner is 1 standard deviation above his mean negative affect) represents a stronger attractor because a one-unit change on the *x*-axis results in a larger change in negative affect—negative affect occurrences away from the set point will induce greater change, moving negative affect more quickly toward the set point. Conversely, the line with the shallower slope (when the male partner is at average negative affect) depicts a weaker attraction. The difference between the slopes is the relative difference in the strength of the attractors. From a theoretical standpoint, we have just demonstrated a topology that depicts affect buffering in close relationships. That is, when a male partner is experiencing increases in negative affect, the female partner tends to yoke in her own negative affect around the set point of low negative affect.

Instead of strengthening or weakening the attractor, a control parameter can also move the location of the set point—the value of negative affect the individual is drawn toward. In this instance, where the line crosses the 0 value on the *y*-axis is moved to a different location on the *x*-axis. For example, in our figure the set point for negative affect when the male partner was at average negative affect was a score of 4 on some scale (indicating the value one would move toward in time); a control parameter could move the set point to 7 on the same scale. Figure 7 illustrates these properties in that the set point for females is slightly higher when the male partner's negative affect is 1 standard deviation above its mean. Taken together, when male partners experience higher negative affect days, the attractor for the female partner's negative affect strengthens and moves to a slightly higher value. Thus, the male partner's negative affect buffers (strengthens the low state), but also slightly pulls up, the value the female partner regulates toward.

Finally, when a control parameter switches a topological feature from an attractor to a repeller, the corresponding line in Figures 4 and 7 would change from having a negative slope to having a positive slope. To better understand the distinction, begin by considering a single value of negative affect along a negatively sloping line but above the 0 value on the *y*-axis. At this point, notice that the *y*-axis predicts a decrease in negative affect, which would move the point down on the *x*-axis, closer to the set point (where the sloped line crosses the 0 value on the *y*-axis). Now consider another value of negative affect along the same negatively sloping line but below the 0 value on the *y*-axis. This time the *y*-axis predicts an increase in negative affect at the next point in time, moving you closer to the set point again. By changing the slope of the line from negative to positive, this pattern reverses—a value of negative affect above the set point would move toward a

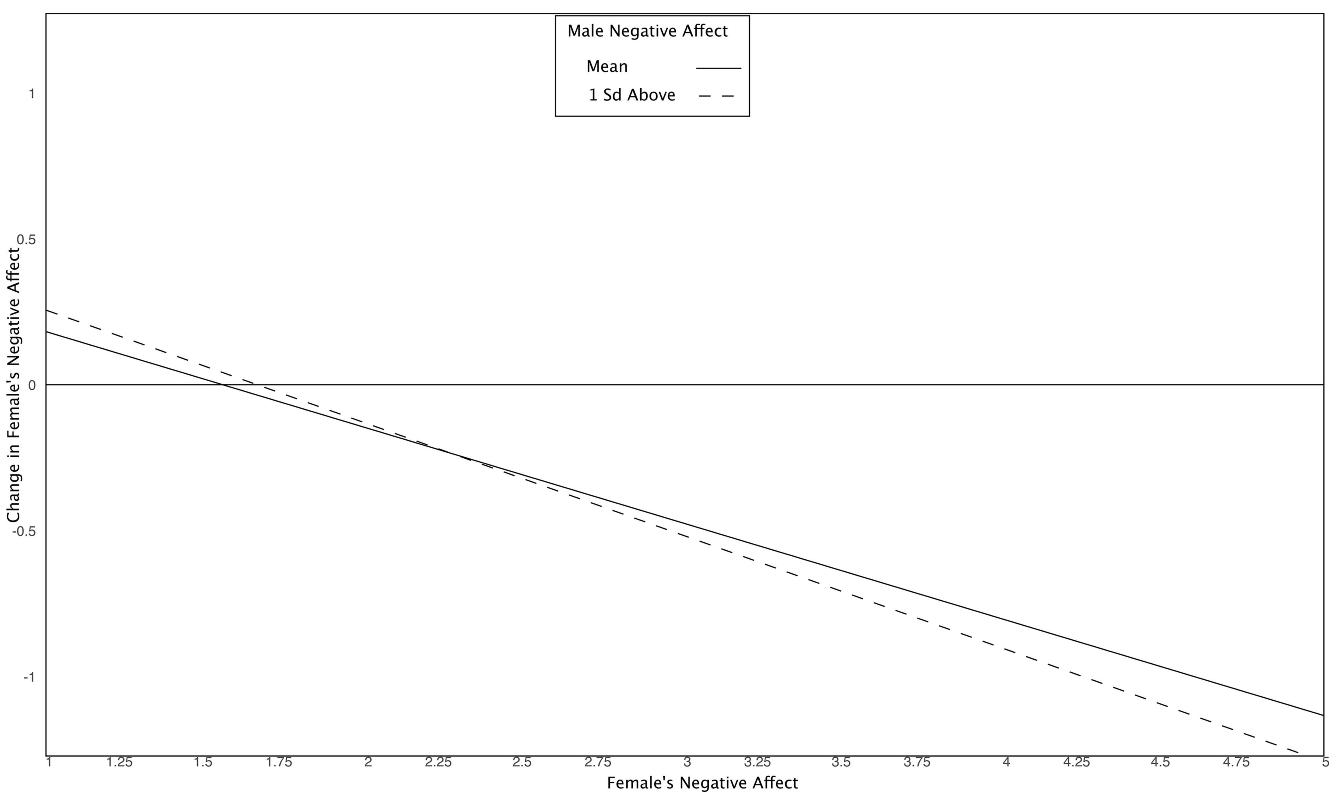



*Figure 7.* One-dimensional state space of female's negative affect with the male partner's negative affect as the control parameter. When males are higher in their negative affect, females' attractor becomes stronger (steeper slope) and increases slightly (higher set point value). This was generated from a multilevel model with change in female's negative affect as the dependent variable and female's negative affect, male partner's negative affect, and a female by male interaction.

higher value of negative affect and a value below the set point would move toward a lower value of negative affect—generating a repeller.

## Two Dimensions

Figure 8 is a two-dimensional state space representation of negative affect and partner's negative affect drawn from a pair of simultaneously estimated multilevel model equations (actor–partner models using change in male and change in female negative affect as outcomes; predictors were group centered to focus on intraindividual change). Here we represent both the partner's negative affect and one's own negative affect through time and looked at the change of both simultaneously. The end result would be the two-dimensional state space with the female partner's negative affect on one axis and the male partner's negative affect on the other, both changing through time. Change in negative affect is represented by the arrows, with the darkness of the arrows representing larger values of change. The model represented here is relatively simplistic for how we might theorize affect coregulation, but is illustrative. Implicit within the state space would be set points that define the location of topological features, the places where we would hypothetically observe no change for the partner's negative affect and one's own negative affect, respectively. In addition, there are areas in the topology where only one outcome is changing at a time; these areas are known as a null cline—equations in *xy* space that represent no change in *x* or no change in *y*. Null clines are critical to converting a state space into testable equations. In the Equations of Change section, we will demonstrate how to go about identifying the null clines and their usefulness.

With a two-dimensional state space there are several possible types of topological features we might observe, but most can be captured as a function of four common ones (Abraham & Shaw, 1992). The four common features are fixed point attractors, fixed point repellers, saddles (or separatrices), and limit cycles. We will continue to use negative affect within couples as an example in demonstrating each of these features. Our goal will be to generate a topology that can represent the regulation of both the male and female partner negative affect simultaneously.

The fixed point attractor is the two-dimensional valley (the set point is the lowest point in the valley) as seen in Figure 8. It would be the value of one's own and one's partner's negative affect that each person is drawn toward. For example, we observe a fixed point attractor of the combination for low female negative affect and male partner negative affect (the 0,0 coordinate in Figure 8).

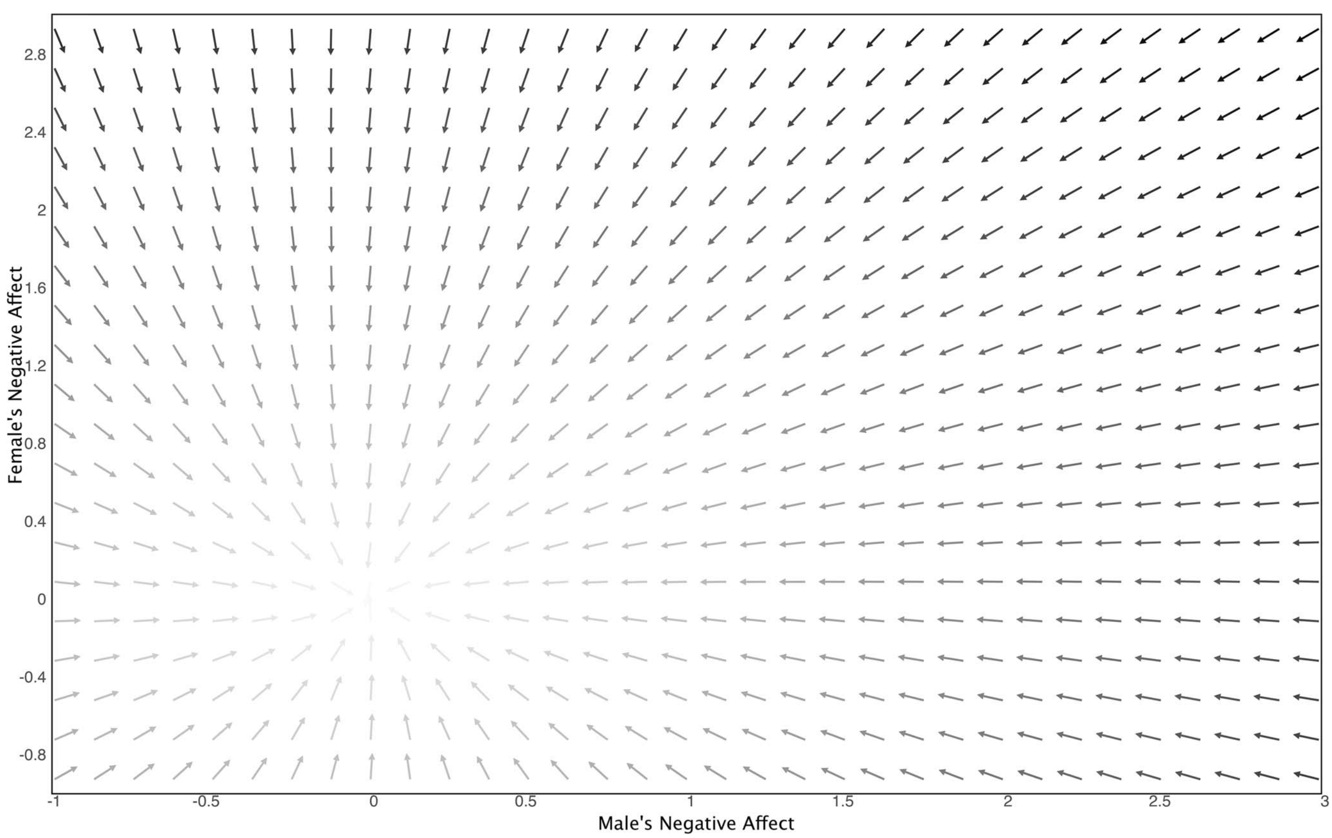



*Figure 8.* A two-dimensional state space of the male's negative affect and female's negative affect where both are changing through time. This depicts a two-dimensional fixed point attractor. Each partner's level of negative affect moves toward a specific point (graphically represented here as 0,0). How quickly a given partner's affect changes depends on how far he or she is from the fixed point attractor.

The fixed point repeller is analogous to a mountaintop, where the set point is the peak. The values of self negative affect and partner negative affect for an individual would always move away from the value represented by the repeller. Low female negative affect and male partner negative affect may function as a fixed point repeller, where the two states are incompatible such that female negative affect or male (or both) negative affect changes.

Saddles can be conceptualized as ridgelines, where they are attractive in one direction and repulsive in another. They often separate the state space into basins of attraction. where each side of the saddle has different attractive properties (e.g., pulled to a different fixed point attractor). For example, there might be a saddle between low female negative affect and low male negative affect as one attractor and high female negative affect and high male negative affect as the other attractor (this description of two attractors with the saddle between them will be the analog to our theory shortly). All of the trajectories on one side of the saddle get drawn toward one attractor, while all trajectories on the other side get drawn the other way. As a result, midlevels of female and male negative affect exhibit the behavior of a repeller, pushing you to one side or the other.

Limit cycles are the equivalent of a looped trail in a two-dimensional space, where the values on some variable continue to repeat, but not necessarily repeat the same value each time. For example, we could imagine a loop between female negative affect and male partner negative affect where we are constantly cycling between low to middle to high and back down again. This is the topological representation of oscillations, in which the set point in the two-dimensional state space is the point we oscillate around.

Combining these elements allows for many other possibilities that expand the utility of topological representations when depicting complex theories. For example, one might theorize the presence of spiral attractors (spiraling toward the set point in time) by combining a fixed point attractor with a limit cycle or the presence of spiral repellers (spiraling away from the set point) by combining a fixed point repeller with a limit cycle. Combinations of the four primary two-dimensional features, in essence, capture the common trails through the topology and afford the possibility for twisting flows of change. Theory dictates what topological features one should expect to find in one's data and the specific values at which these features are found.

## An Example of Two-Dimensional Translation

We can now go through the process of translating theory to topology on what we might expect between male and female negative affect in couples. The results in Figure 9 come from equations where we only allowed a single topological feature and are thus not surprising in that at its simplest, we would expect the pattern of negative affect over time to move toward a state of low–low negative affect (average negative affect was close to the low end of the scale). Couples themselves are often described as the ultimate form of social support for dealing with problems (see Baucom & Eldridge, 2013, for a review). Couples utilize several forms of accommodation to help one another, but also help themselves (Rusbult, Verette, Whitney, Slovik, & Lipkus, 1991). Together these imply an attraction toward managing problems as they arise—a verbal description of an attractor.

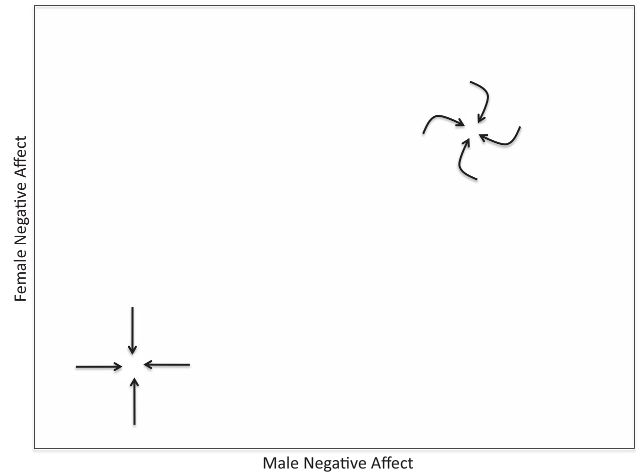



*Figure 9.* An incomplete translation of the running negative affect and mood example into a hypothetical topology. The figure includes only vectors around the hypothetical topological features. Since topologies are assumed to be smooth, the features imply the areas between them as well.

However, this is an overly simplistic view of couples in that there is also ample literature on distress in couples. As an example, distressed marriages can generate reciprocally negative interactions that feed one another through time (Margolin, 1981). Such descriptions imply a second attractor that can occur at high–high negative affect. Furthermore, the interchanges of negative responses resulting in negative responses from the spouse suggest spiral-like properties. A model that would account for both of these circumstances would thus have two attractors instead of one where the second attractor was a spiral attractor. Figure 9 is a simple translation of this written description.

The theory laid out thus far implies an additional topological feature too. Each of the elements depicted in Figure 9 is an attractor that must have a saddle or repeller that defines the boundary between each other. So we must imply some saddle or repeller that separates out the two domains. Thus, our final representation of the theory as an expected topology can be seen in Figure 10, where we include all three primary topological features.

To translate this theory into a testable model, we need merely draw lines that cross at all the set points. These lines are the hypothetical null clines (more on this in the Equations of Change section). All the topological features must be connected by the lines (two lines, one for each dimension). The equation forms for those lines specify the equation forms that need to be tested to generate the hypothesized model. Figure 10 includes the null clines crossing at each of the three set points. Our way to connect the three set points implies a pair of cubic equations (one of the lines could have been linear, but this would have forced a symmetry between males and females, which was not the case, as shown below).

To test out theory, we generated a multilevel model using an actor–partner structure predicting the change of male and female negative affect as the dependent variable and the two equations implied by the hypothetical null clines as the predictors. This particular model fit our data very well, generating a pseudo-$R^2$ of .41 (calculated by sum of the squared predicted values over the

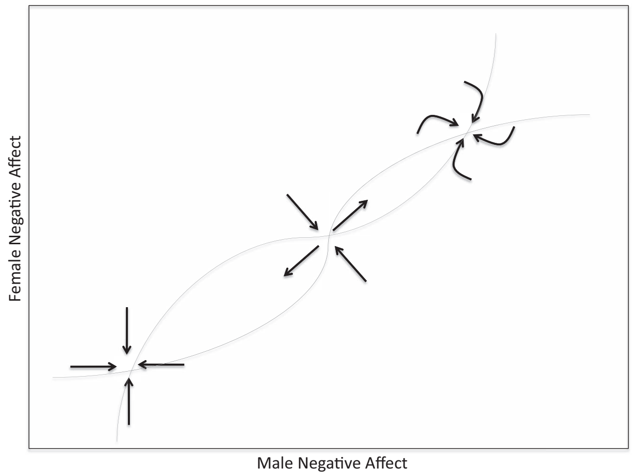



*Figure 10.* A translation of the running negative affect and mood example into a hypothetical topology with the inclusion of saddles that separate each basin of attraction and a repeller where the saddles meet. Two null clines (one for each dimension) represent areas where a single dimension does not change. The points where the two null clines cross represent set points where neither of the variables is likely to change.

sum of the squared predicted plus squared residuals). However, the results were not exactly as expected. Figure 11 is a state space made from these results. Specifically, we observed the low–low attractor and we observed a saddle, but the saddle was not where it was expected. Furthermore, there was suggestion of a second attractor, but it was not observed in the data (beyond the data range).

Examining the divergence of our theory-driven topology from the equation-estimated topology helps inform where our theory is lacking—as a good tool should. Let us examine each of these unexpected results independently. Our lack of the second attractor may be due to the fact that the examined couples all scored highly on relationship satisfaction and thus may be a limitation of our ability to generalize the sample (see Butner et al., 2007). However, this leads to a possible clue of where to go next in that the repeller is barely within the data range, creating a stickiness or slowdown of trajectories near the ridgeline (notice that the change is slower, represented by lighter arrows, around the saddle at high female negative affect). Kelso (1995) argued that stickiness in time series can be indicative of an attractor that is only sometimes there. We could be missing a control parameter that moderates the existence of the second attractor—possibly relationship satisfaction. For example, we might expect that when relationship satisfaction is high, only the low–low state exists. When relationship satisfaction is low, instead we might expect the second attractor to strengthen, implying times where the couple is in the low–low basin and times where the couple is in the other basin. It might even be possible for only the non-low–low basin to be the only stable attractor for some couples when the relationship is truly on the rocks. This distinction is not unlike the types of supporters identified in the social support literature—supportive, ambivalent, and aversive (Uchino, Smith, Carlisle, Birmingham, & Light, 2013). Since our tested model did not allow the topology to differ by other theoretical variables (or even across couples), allowing for a changing map is one obvious course of investigation.

The lack of symmetry between men and women is also intriguing. The results suggest instead that men in the sample were always benefiting from the couple's relationship—being pulled toward low negative affect—while there was a cut point (the saddle) for females where they no longer benefited from the couple. This is consistent with the body of literature suggesting that there is an asymmetry between men and women in terms of who does more and who benefits from relationship maintenance (Ragsdale, 1996).

From these results, there immediately become implications of where we would need to go next—consider control parameters that moderate the existence of the second attractor with a more diverse sample to do so and also to consider how the asymmetry between males and females in the relationship would potentially impact the theory and resultant pattern. Most importantly, topological maps are relatively simple graphical representations of a limited set of equations, allowing us to visualize and test our theory even when it can be quite complex. The advantage that topologies have over many current approaches is that they facilitate the translation from theory to statistical model and highlight our lapses, suggesting new directions and integrations. We just learned the steps to translate a theory into a topology, and now we will focus on how to translate the topology into equation form.

## Equations of Change

We have made an argument for why we need a tool like topology, described how topology works, and described how to translate a theory into a topological representation. The goal of this section is to demonstrate how to translate a given topology into an equation or set of equations of change. For clarity, we will express the equations where the derivatives are the outcomes. This is a direct analog to the implied equations from LDE (see Boker et al., 2003) and LDS (though difference score modeling implies discrete time; see McArdle, 2001). Given that change as an outcome can be translated relatively easily into an autoregressive relationship (Huckfeldt, Kohfeld, & Likens, 1982), we also believe this applies to DFA approaches (see Wood & Brown, 1994) and the general use of lag relationships (though we will not spend time on translating topology to a lag equation form).

LDE, LDS, and DFA models can be quite complicated to build, but the equations they imply, when expressed in terms of derivatives, are relatively simple. In fact, we will see that topology from theory often implies models untested in our modern methods. Since methods now exist that allow for direct approximations of derivatives that can be used in regression, multilevel modeling, and structural equation modeling (see Deboeck, 2010), we will use basic regression notation using unstandardized coefficients.

Topologies necessitate the use of nonlinear regression equations. There are multiple ways to do this (e.g., trigonometry, nonlinear transforms), but we will stick with a polynomial and interaction combination because it is flexible, familiar, and well explored. For the purposes of the equations, we will ignore errors of estimation and the need to include lower order terms when examining higher order terms. This is done to focus our attention on the model itself and should be translated accordingly (i.e., lower order terms should be included with an eye toward the impact of centering and scaling). Given that these are regression equations,

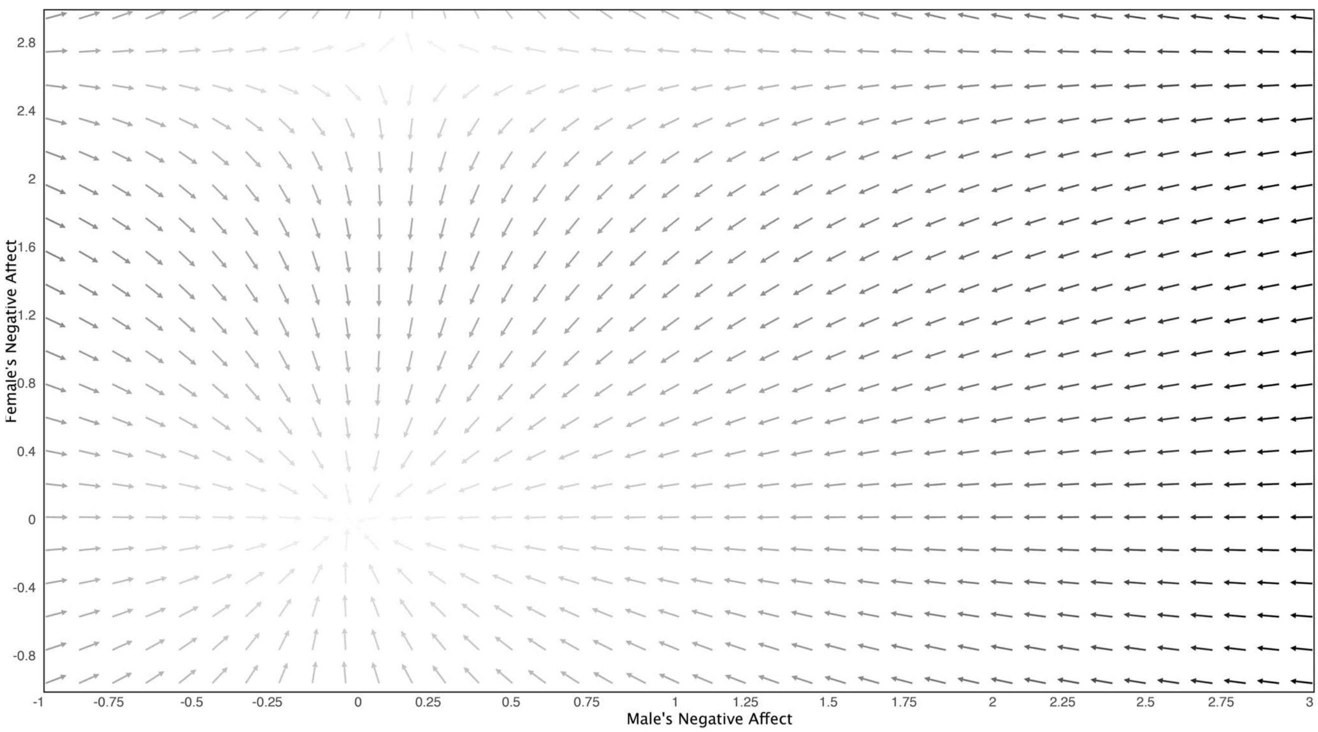



*Figure 11.* Topology generated from the estimated equations extracted from our hypothetical state space. There are two main divergences from our theory. The second attractor indicative of problematic cycling in the couple was beyond the data range, and the saddle was asymmetrical, showing cut-point-like behavior for females (high negative affect going higher) but not males.

they should follow the standard rules for regression (see Cohen, Cohen, West, & Aiken, 2003).

## One-Dimensional Structures

A single equation where change is the outcome generates a one-dimensional topology with or without control parameters. We will start overly simplistic where the equation only has an intercept to show what these models are reduced to if they fail to predict:

$$\frac{dx}{dt_{it}} = b_0. \tag{1}$$

In Equation 1, change is depicted as constant. When a regression is run with no predictors, the intercept takes on the average value of the dependent variable and, in this case, average change. This is akin to linear growth models that depict constant change. For example, this equation would capture the average change in negative affect over time.

To begin to capture topology, we need to add the variable used to represent change into the equation:

$$\frac{dx}{dt_{it}} = b_0 + b_1 x_{it}. \tag{2}$$

Once ($x$) has been added as a predictor of change in $x$, we have an equation form that can represent a topology. Specifically, the equation implies a set point where no change would occur. The behavior around that set point defines the topological feature as an attractor or repeller. To identify the set point, we set change to equal 0 (the dependent variable) and solve for the value of $x$ (the value of $x$ when change is 0). In this particular equation the set point is at $-b_0/b_1$. To identify what kind of topological feature it represents, we look at the sign on the slope of $b_1$. Remember that when the slope is negative, the equation implies an attractor. When the slope is positive, it implies a repeller. Notice that this is the direct analog to the scatterplot shown in Figure 5.

Consider an example from King et al. (2006) where change in posttraumatic stress disorder (PTSD) scores was modeled as a function of previous PTSD using latent difference scores. They hypothesized a single set point (which comes across in their equations rather than from drawing a topology) and found an attractor: ${}^{dx}/_{dt} = 15.31 - 0.803x_{it}$. In order to calculate the set point, one must divide the inverse of the constant change (i.e., $-15.31$) by the average slope (i.e., $-0.803$), which results in a value of 19.07. At a value of 19.07, we would expect to see no change in PTSD scores. Furthermore, the slope of $b_1$ is negative, suggesting that 19.07 is an attractor. According to the tested equation, over time individuals should move toward a value of 19.07 on the PTSD scale.

When $b_1$ is positive, we instead observe the pattern of a fixed point repeller where cases are pushed away from the set point, also defined at $-b_0/b_1$. In reality, observing a repulsive set point should be rare. We would only ever observe flows away from the repeller because systems are thought to be constantly perturbed (small changes can occur from parts of the system not being modeled, such as lower and higher order parts of the system) and a slight change off the set point pushes the case away from the fixed point repeller (Stewart, 2002). It is also possible that if the timing of measurement missed the flow of a variable away to some other topological location, then there would never be a representation of the repeller in the data. As noted in our theory to topology translation, repellers are often implied even when they are not observed as the borders between attractors.

The rate of change just off the set point captures the strength of attractiveness or repulsiveness. In this case, the slope is constant at all levels of *x*, capturing the strength of attractiveness or repulsiveness. If we ignore the sign (indicating whether it is an attractor or repeller), the slope dictates how quickly one would move toward or away from the set point. So, a steeper slope is indicative of a stronger topological feature (e.g., a rolling hill vs. mountain cliff). This numerical value is known as a local Lyapunov exponent (Kantz & Schreiber, 1997) or characteristic root (Abraham & Shaw, 1992).

The Lyapunov exponent is a numerical value that captures the rate of entropy for a given topological representation of the data (i.e., the steepness of the slopes that make up a valley or a hill). The Lyapunov exponent in King et al.'s (2006) examination of PTSD over time is the average proportional change, or −0.803 units on the PTSD scale. The larger the value, the more quickly the PTSD scores will settle on the set point over time.

Let us consider the impact of other variables in Equation 2. Adding other variables as predictors captures the potential impact of control parameters because they alter the topological features. In our one-dimensional negative affect example, adding partner's negative affect as a predictor had the potential to alter the topological feature, but whether we add the predictor as a main effect or interaction has dramatically different effects. If a predictor is added as a main effect, it can only influence the set point location. Predictors that are allowed to interact with each other can alter both the location of the set point and the strength of the set point (as actually occurred in Figure 7). In Equation 3, we add another variable (*a*) as a main effect on change:

$$\frac{dx}{dt_{it}} = b_0 + b_1 x_{it} + b_2 a_{it}. \qquad (3)$$

Following the same rules as before, we can solve for set points by setting change to 0 (the dependent variable) and solving for *x*. Now we observe that the set point is a function of the control parameter *a*:

$$x = \frac{b_0 + b_2 a_{it}}{-b_1}. \qquad (4)$$

To identify the Lyapunov exponent, we need to identify the change that occurs just off the set point (at the limit of the set point). To do this we take the derivative of Equation 3 with respect to *x*. This can be confusing because the dependent variable is a form of derivative. The easiest way to understand this is to imagine the dependent variable as another variable entirely (e.g., *U*), just like a standard regression analysis. In Figure 12, we show a slightly curved equation relationship between the dependent variable and *x*. Taking the derivative with respect to *x* (as opposed to "with respect to time") identifies the slope of the tangent line near the intersection between the function and where change is 0—which is the local Lyapunov exponent. The derivative of Equation 1 equals a value of 0; there is no topological feature. The derivative of Equation 2 equals the slope, $b_1$. The derivative of Equation 3 is also equal to the slope, $b_1$. Adding in main effects to this equation moves the set point but does not alter the type of topological feature or its strength.[3]

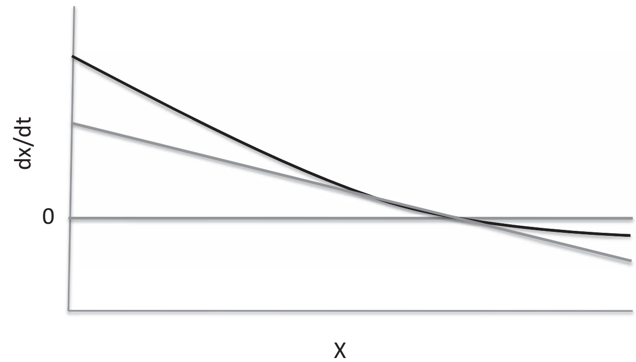



*Figure 12.* A one-dimensional state space including the dimension of change on the *y*-axis. The Lyapunov exponents for a topological feature are the tangent lines at each set point. The equation for the Lyapunov exponent expresses the topological forces at any value of *x*.

For a variable to alter the type and strength of a topological feature in a one-dimensional state space with only a single set point, there must be an interaction with *x*. Equation 5 adds the same control parameter but this time as an interaction with *x*:

$$\frac{dx}{dt_{it}} = b_0 + b_1 x_{it} + b_2 x_{it} a_{it}. \qquad (5)$$

In Equation 5, (*a*) is capable of influencing the value of the set point, the type of topological feature, and its strength. Solving for the set point (setting change, the dependent variable, to 0 and solving for *x*), we get $x_{it} = -b_0/(b_1 + b_2 a_{it})$, a value that is different as a function of a. Solving for the Lyapunov exponent by taking the derivative of Equation 5 with respect to *x*, we get $b_1 + b_2 a_{it}$. Notice that the value of the Lyapunov exponent differs as a function of *a*.

Also notice that adding *a* as a moderator can make the Lyapunov negative or positive, allowing for an attractor at some values of *a* and a repeller at others. The limitations of what a does are a function of the scaling of *a* and *x*, respectively. The key point here is for a variable to alter a topological feature beyond just moving the set point—it should be treated as a moderator. This gets more complicated when *x* takes on polynomial forms, but the idea is the same.

The equation form thus far is capable of representing a topological feature and allowing for another variable to alter this topological feature. Topologies, and their underlying equation forms, provide researchers with more precision and flexibility

[3] Given that most phenomena are measured on a limited discrete metric, it is possible for a control parameter to essentially move a set point outside the data range or even scale range. This is essentially a form of emulating a flow or extinguishing/changing a topological feature due to measurement limitations rather than capturing it in equation form.

when translating between theory and model testing, in part because topological features can be altered, but also because there can be multiple topological features. In the example of negative affect, it is possible to have more than one stable level of negative affect, as observed in the particular participant in our data set in Figure 6. To generate the equation for multiple topological features, we need to explore nonlinear relationships between change in *x* and *x* as a predictor. Let's start with a simple case of $x^2$:

$$\frac{dx}{dt_{it}} = b_0 + b_1 x_{it} + b_2 x_{it}^2. \quad (6)$$

Figure 13 shows a best fitting line of hypothetical data representing *x* squared. Notice that the line crosses the point where change in *x* is 0 at two values of *x*, giving us two set points. Furthermore, we can now imagine the Lyapunov exponents as tangent lines at each of the set points (drawn as light gray lines on Figure 13). One of the slopes is negative, indicating an attractor, while the other is positive, indicating a repeller.

Taking the derivative of Equation 6 with respect to *x* and treating the dependent variable like it is a different variable (e.g., *U*), we can identify the value of the Lyapunov exponents. Unlike the linear equation form, the resultant equation ($b_1 + 2b_2x_{it}$) is a function of *x* itself. That is, we generated a characteristic equation of how forces differ at varying parts of the map. Since Lyapunov exponents are best understood at the topological features (e.g., set points), we can stick in the value of the set points for the value of *x* to identify each Lyapunov or the strength of each set point.

As a comparison, let us generate the same graph for a cubic relationship:

$$\frac{dx}{dt_{it}} = b_0 + b_1 x_{it} + b_2 x_{it}^2 + b_3 x_{it}^3. \quad (7)$$

The accompanying phase space is shown in Figure 14. Now there are three values of *x* where the change in *x* is 0—three set points.

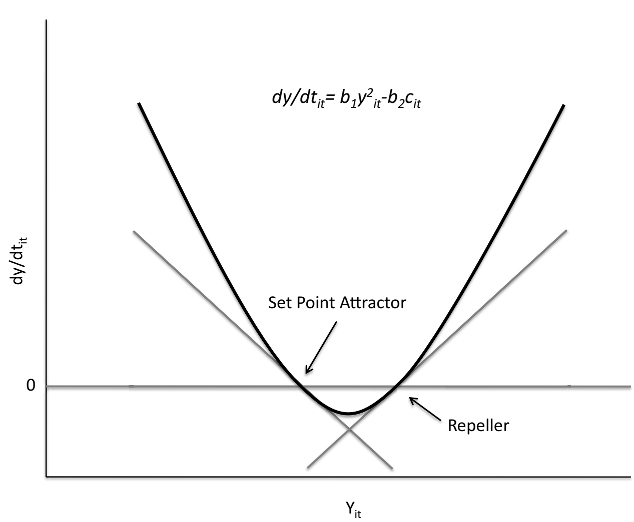



*Figure 13.* Graph in scatterplot form (where change is on the *y*-axis and *y* is on the *x*-axis) of the one-dimensional state space for a quadratic relationship between *y* and change in *y*. The light gray lines show the Lyapunov exponents around each set point.

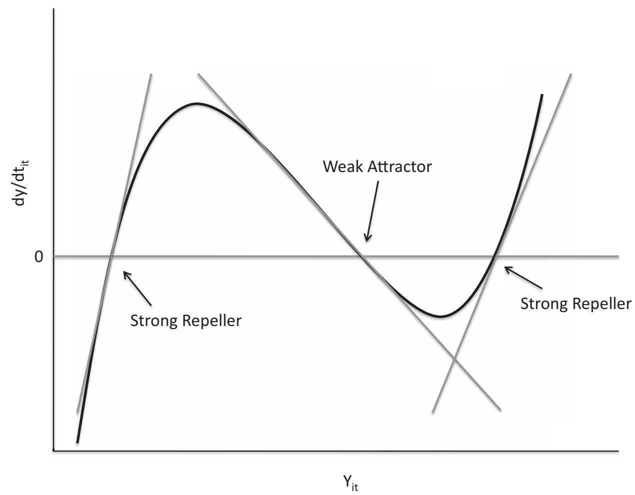



*Figure 14.* Change is on the *y*-axis and *y* is on the *x*-axis. The graph depicts a one-dimensional state space for a cubic relationship between *y* and change in *y*. The light gray lines show the Lyapunov exponents around each set point, showing the repellers (positive slopes) and an attractor (negative slope).

There is a direct relationship between the degree in the polynomial form and the number of possible set points. It is also clear that the set points switch off between being attractive and repulsive. This is due to the fact that in order to separate two or more attractors, a repeller or saddle must define the boundaries of each attractor.

As our last example in one-dimensional topology, let us combine the idea of moderators and polynomial relationships exploring the cusp catastrophe, a relatively well-known equation in mathematics (Guastello, 2011). The cusp catastrophe model has been applied several times in psychology, from areas ranging in binge drinking with attitudes toward alcohol to speed–accuracy tradeoffs and a general model of attitudes (Dutilh, Wagenmakers, Visser, & van der Maas, 2011; Smerz & Guastello, 2008; Latané & Nowak, 1994), but was originally applied by Lorenz (1966) to understand emotional expression in dogs. The cusp catastrophe includes a cubic function of *x* and two control parameters (one as an interaction):

$$\frac{dx}{dt_{it}} = b_1 x_{it}^3 + b_2 x_{it} c_{it} + b_3 a_{it}. \quad (8)$$

The cubic relationship will allow for three set points, and in the common catastrophe model two of the set points function as attractors while the third functions as a repeller between the two (as before, we can now solve for the set points and local Lyapunov exponents for each). The existence of these two stable states is a function of the control parameters *c* and *a*. The control parameter *a* functions as an asymmetry term, altering the strength of attraction for each of the two attractors, while *c* can strengthen or extinguish the stable states via the bifurcation, turning the three set points into a single set point (see Figure 15 as an example).

## Going From Hypothetical Topology to Testable Equations in One Dimension

In the one-dimensional topology case, generating the topology from theory and into equation form is relatively easy. All the

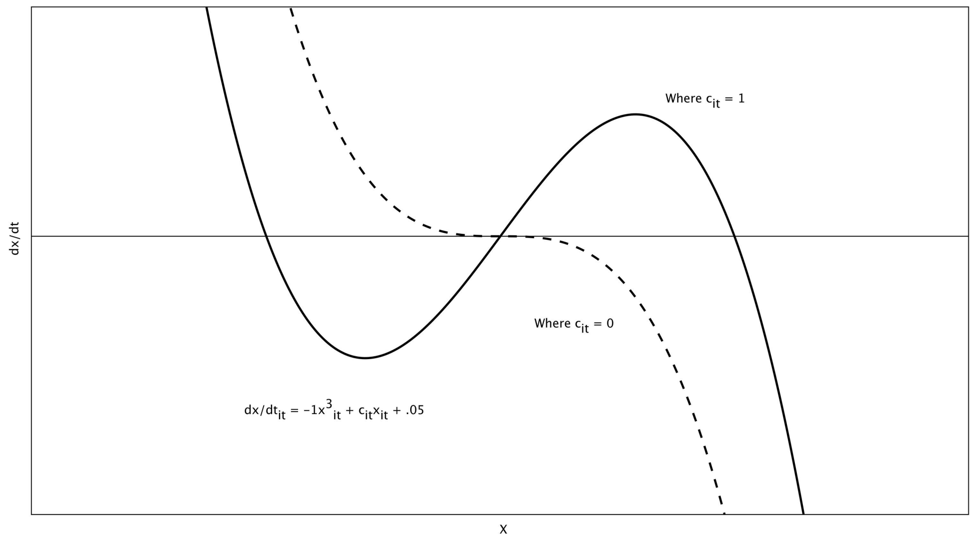



*Figure 15.* Example one-dimensional state space from the cusp catastrophe model with two values for the control parameter *a* using Equation 8. The solid line has three set points (one repeller and two attractors). The dotted line has only one set point, an attractor. This ability sometimes to have one stable state versus two is due to the inclusion of higher order interactions in terms of *c* and the polynomial of *x* (*a* can also turn the model into having a single set point by making the cubic function extremely asymmetrical).

equation representations can be thought of in terms of the scatterplots with change on the *y*-axis and the same variable on the *x*-axis. Generating set points merely requires identifying the places on the *x*-axis one would expect 0 change. Attractors need repellers between them to identify the regions of attraction or borders between the attractors. Finally, one need merely draw a hypothetical line connecting the set points where the slopes are negative at the set points for the attractors and positive for the repellers.

The equation for the line that connects the set points while maintaining the proper signs on the slopes at each set point is the equation to be tested. For example, the individual in Figure 5 implies a linear equation: change in negative affect$_t$ $= b_0 + b_1$(negative affect$_t$). Our individual in Figure 6 implies a cubic equation: change in negative affect$_t$ $= b_0 + b_1$(negative affect$^3_t$) $+ b_2$(negative affect$^2_t$) $+ b_3$(negative affect$_t$). To argue that these two individuals reside in the same topology would require either the cubic form (assuming that the first person never left the first attractor but could with the right perturbation) or an equation form that includes a control parameter to dictate when we would observe one model versus another. This control parameter model could be a case of a linear form (like the first person) with a main effect as a function of the control parameter that can move the set point (in this case, the control parameter is changing within the second person's data so that the location of the attractor itself is sometimes high and sometimes low) and an interaction (because the strengths of attraction are not equal for that individual). This is exactly what we tested to produce the Figure 7. Or it could be something closer to the cusp catastrophe model where control parameters sometimes allow for unistable patterns and other times allow for the multistable one.

Each model is different from the other, requiring careful thought on theory enhancing the theory generation process. Each circumstance implies different testable results—the catastrophe model suggests that, all else being equal, there can be two stable states (though the existence of the stable states are moderated), while the linear interaction model suggests that there must be something that moderates the high stable pattern versus the low stable one—multistability is not possible. In the end, the process itself provides utility for enhancing theory.

## Two Dimensions in Equation Form

To move to a two-dimensional topology where we look at the outcome on two variables, we need to consider two equations of change simultaneously. Each equation treats change as the outcome—the hidden dimensions of the topology. As before, we can identify set points and Lyapunovs. However, set points and Lyapunovs are now the function of both equations. For example, let us begin with two linear equations of change using *x* and *y*:

$$\frac{dy}{dt_{it}} = b_0 + b_1 y_{it} + b_2 x_{it}, \qquad \frac{dx}{dt_{it}} = b_3 + b_4 x_{it} + b_5 y_{it}. \tag{9}$$

This model is known as the bivariate model in latent difference scores (McArdle, 2001). It includes a pair of crossover influences captured by $b_2$ and $b_5$, respectively.

Using an advanced mathematical graphing tool is the easiest way to reconstruct a topology from an equation for a two-dimensional state space. Many of our graphs utilized Grapher (a program that comes preinstalled on all Apple computers running OS X). However, we will also need to understand a known hand method because it is key to converting two-dimensional topologies from theory to equations. The key to decomposing two outcomes in time (e.g., finding the set points and Lyapunov exponents) at the level of detail discussed herein is to draw upon null cline approaches (Gottman et al., 2002).

Null clines depict the coupled equations in *xy* space when the changes (i.e., dependent variables) are fixed to 0. In essence, they are the expansion of set points to two dimensions, making a pair of lines instead of points. To identify the null clines, begin by considering the circumstance where each equation would show no change—a velocity of 0. For this we consider each equation in isolation in a *xy* space. In both cases we solve for *y* to place it back into regression equation format. The first equation in Equation 9 would generate a relationship of $y_{it} = -(b_0 + b_2 x_{it})/b_1$, and the second a relationship of $y_{it} = -(b_3 + b_4 x_{it})/b_5$. Each of these null cline equations, when graphed, represents lines where no change occurs on that dimension. The set points of the system are where these two lines intersect, and each area sliced by the two equations can potentially have different patterns of change.

Figure 16 shows the null clines overlaid on the male–female negative affect state space in Figure 8. As mentioned earlier, this state space was generated by simultaneously treating the change in negative affect for males and the change in negative affect for females as the dependent variable using an actor–partner-style model. In this case our tested equations were the same as Equation 9. The two null clines only cross once at the set point for the attractor.

To go from theoretical topology to testable equations, this is as far as one needs to understand about null clines (the lines represent no change in one variable and generate set points where they cross). However, they can be used to identify the set points and the Lyapunov exponents. We illustrate the complete process using Littlefield, Vergés, Wood, and Sher (2012). It generates a single attractor similar to our midstep negative affect example (where we only allowed for a linear equation).

Littlefield et al. (2012) modeled the relationship between novelty seeking and alcohol consumption of college students using three waves of data over their 4 years in school; they presented some evidence that novelty seeking predicted changes in alcohol use. For our purposes, we will restrict our discussion to their first model (their Figure 1a), though they tested some interesting alternatives. For illustrative purposes, we will discuss the model in terms of change over the eight semesters. While these authors utilized a form of LDS model that has similarities to Equation 9, we will interpret their results using Equation 9. This model (along with the others tested) only provided variance/covariance parameters lacking information necessary to estimate any intercepts. We therefore treat the intercepts extracted from their equations as 0. Having different intercepts would move the location of the set point, but not alter the map in any other way.

The null clines (as illustrated in Figure 17) were identified by solving the equations, ignoring the other equation entirely. First, we infer that Littlefield et al. (2012) considered the linear change in heavy drinking to be 0 units (i.e., $b_0$; our best guess from the exclusion of mean structure in the model) and for this to change proportionally by $-0.50$ (i.e., $b_1$) for each unit of heavy drinking and by 0.01 (i.e., $b_2$) for each unit of novelty seeking. For changes in novelty seeking, the authors fixed the linear change to be 0 units (i.e., $b_3$; again, our best guess), the proportional change predicted by novelty seeking to be $-0.37$ units (i.e., $b_4$), and the proportional change predicted by heavy drinking to be 0.04 units (i.e., $b_5$). Therefore, we computed the null clines, placing heavy drinking on

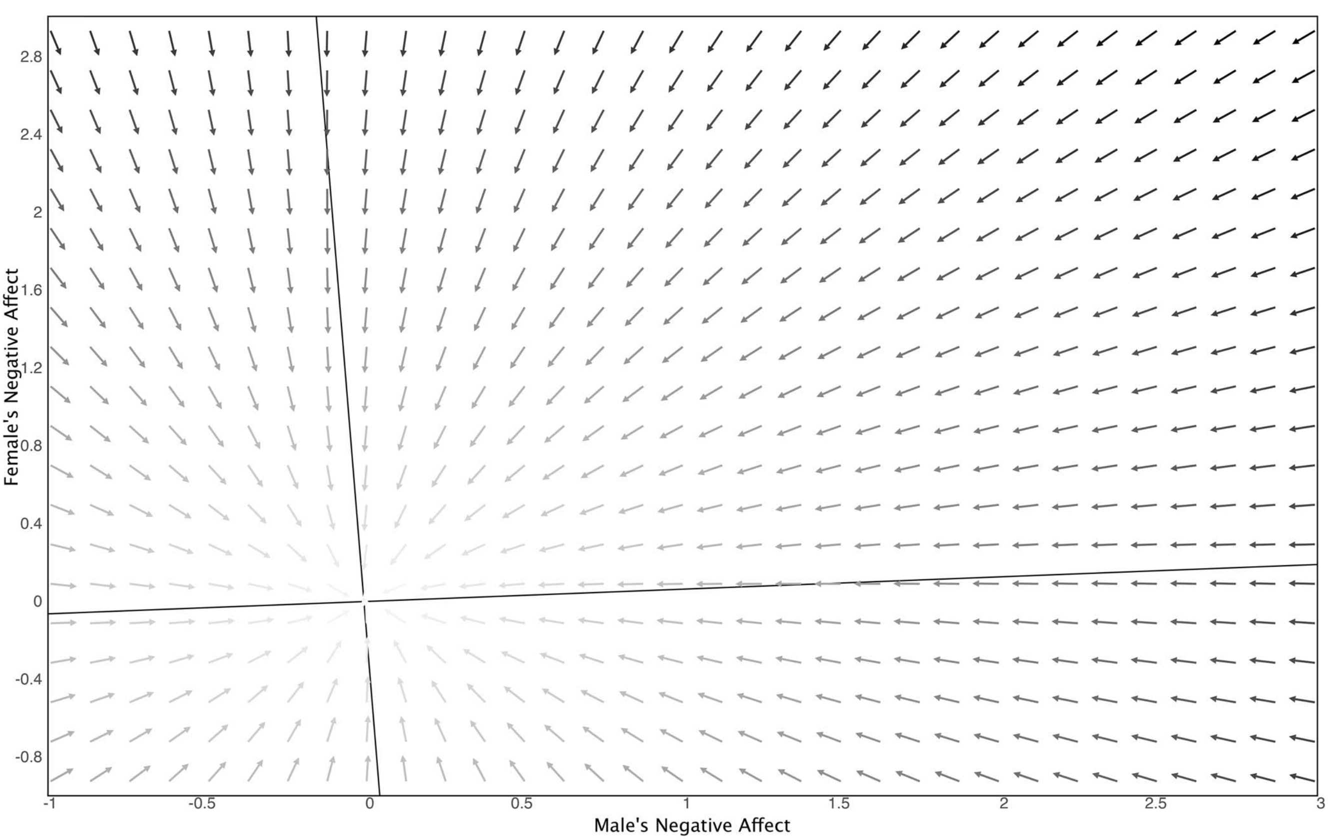



*Figure 16.* The two-dimensional state space of male and female negative affect from a pair of coupled linear equations with the inclusion of null clines. These equations were the same as Equation 9.

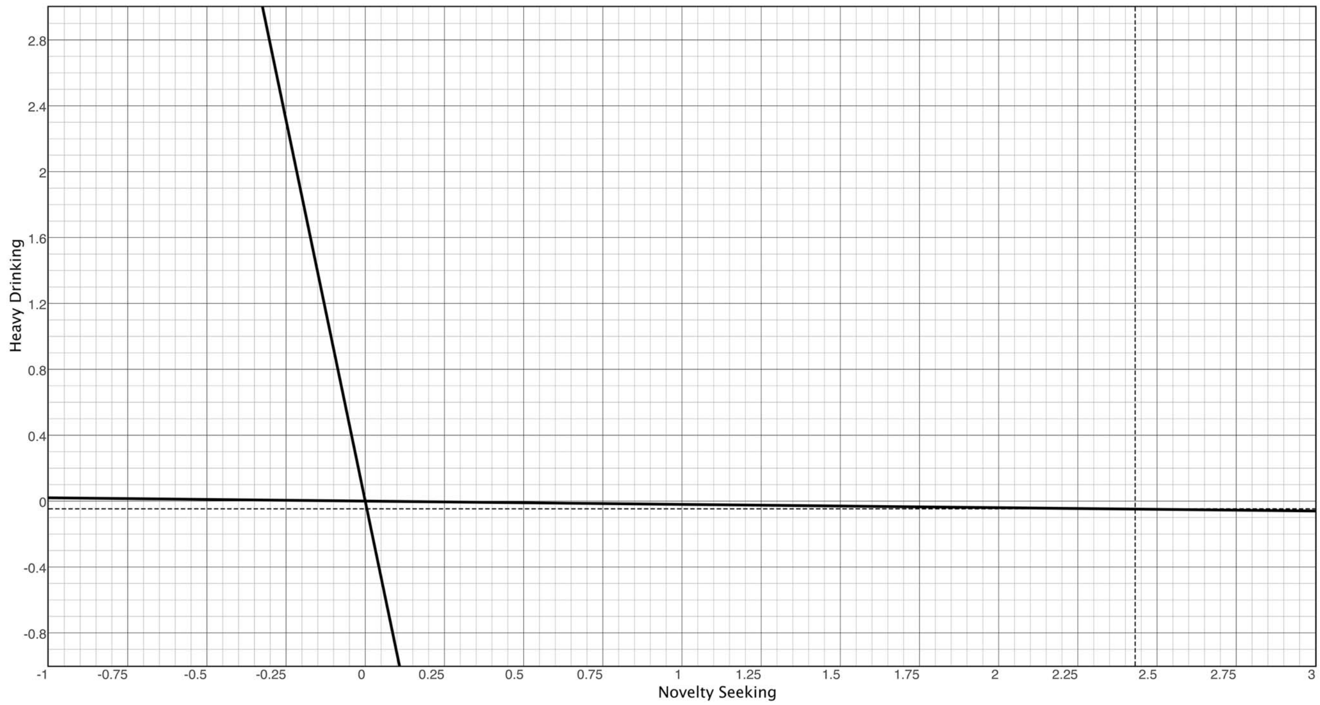



*Figure 17.* Null clines generated from the Littlefield et al. (2012) data. Their results imply a single set point where the two lines cross. This splits the topology into four territories each of which can have different behavior. The set point is at 0,0 because of our inference for the intercepts that the model did not specify.

the *y*-axis and novelty seeking on the *x*-axis, as having null clines of heavy drinking = −.02(novelty seeking) and heavy drinking = −9.25(novelty seeking). This implies a single two-dimensional set point at 0 novelty seeking and 0 drinks (the point the two lines cross), though the location of this set point is completely determined by our inference of intercept values. One way to think of this 0/0 location is as the mean value (logically true with a single topological feature, though not always true).

This slices up the topology into four areas, where an individual has more than 0 drinks and more than a 0 in novelty, where an individual has more than 0 drinks and less than a 0 in novelty, where an individual has less than 0 drinks and more than a 0 in novelty, and finally where a person has less than 0 drinks and less than a 0 in novelty. The general behavior of each quadrant is identified by taking an exemplar value of heavy drinking and novelty seeking within each area and using the original equations to determine the velocities (where it goes). It is not even necessary for these points to be close to the set point (though going too extreme can overly simplify the description of the quadrant's behavior). For example, if you insert eight drinks and a score of 2 on novelty seeking into the original equations, the derivatives give a sense of the behavior for the entire "more than 0 drinks and greater than 0 novelty" quadrant. In this case we would get heavy drinking = −0.5(8 drinks) + 0.01(2 novelty seeking) and novelty seeking = −0.37(2 novelty seeking) + 0.04(8 drinks). The predicted derivatives would be −3.98 and −0.42, respectively; meaning that if a person were to report consuming eight drinks and score a 2 on novelty seeking, we would predict the individual to decrease his or her number of drinks consumed by 3.98 and decrease his or her novelty seeking score by 0.42 over time. This quadrant moves toward the set point in terms of novelty seeking and heavy drinking (attractive in both dimensions).

Repeating this process for all quadrants derives the overall topological behavior. If you start with high levels of heavy drinking and high levels of novelty seeking, you would see a sharp decrease in your heavy drinking but a slight decrease in novelty seeking over time. If you start with low levels of novelty seeking and high levels of heavy drinking you will decrease your heavy drinking to around 0 drinks (i.e., the set point) and slightly increase your novelty seeking over time. Alternatively, if you start with low levels of both heavy drinking and novelty seeking, you are likely to slightly increase your heavy drinking to 0 drinks and increase your novelty seeking behavior. Lastly, if you start with low levels of heavy drinking and high levels of novelty seeking, you will drastically decrease your novelty seeking and slightly increase your heavy drinking over time. In essence, this is describing a two-dimensional attractor as illustrated in Figure 18.

In a two-dimensional system, each topological feature has two Lyapunov exponents, one in terms of *x* and one in terms of *y*. To identify the characteristic equations, we merely need to examine each equation on its own (as with the null clines) and take the derivative of the equation with respect to the variable whose change we are predicting. This is the same process as was done in the one-dimensional models, except now done for both equations. For example, for the equation where change in heavy drinking is the criterion, we take the derivative with respect to heavy drinking (treating change in heavy drinking as a different variable; e.g., *U*). This results in a value of −.5. For the equation where change in

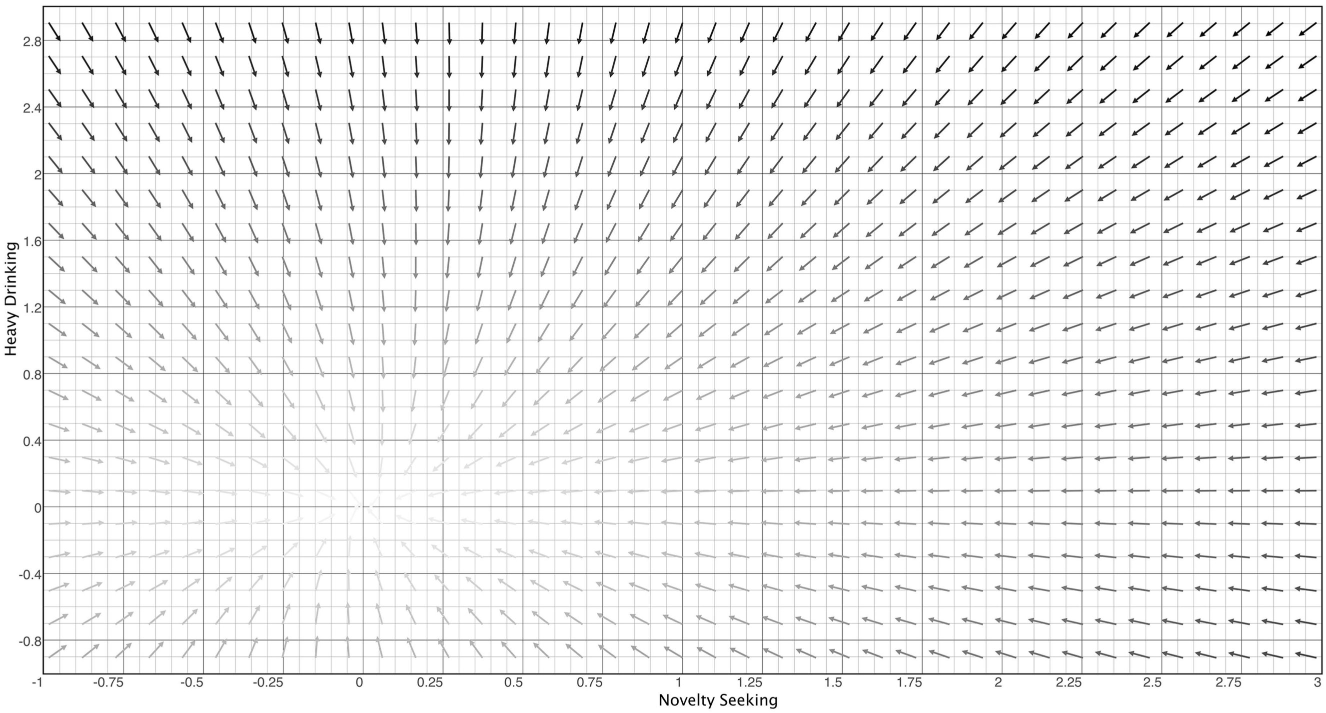



*Figure 18.* Two-dimensional state space of Littlefield et al. (2012) implied by our inferred equations from Littlefield et al.'s results for Model 1.

novelty seeking is the criterion, we take the derivative with respect to novelty seeking (treating change in novelty seeking as a different variable: e.g., *U*). This results in a value of −.37. The set point values for *x* and *y* are then inserted into the equations, identifying the local Lyapunov exponents (since the resultant equations are constants in the Littlefield example, this step is unnecessary—the Lyapunovs are the same over the entire spectrum of values if these equations are correct). This method is imperfect in that it will identify the overall attractiveness and repulsiveness of each dimension (*x* and *y*, respectively) but will fail to depict any oscillatory (angular) effects. These require the crossover or coupling relationships, and the math is complicated in that an oscillatory effect has an imaginary Lyapunov exponent that will come out as 0 under estimation procedures that only consider real numbers (a value of *i*, mathematically speaking). To account for possible oscillatory relationships, one would need to take the eigenvalues of the set of coefficients (estimated at the set point) instead (this matrix is known as the Jacobian matrix). Figure 19 shows how combinations of Lyapunov exponents differentiate the two-dimensional typologies (we add in the last row what happens when using two second-order equations, as discussed below). Combining the Lyapunovs with identifying the patterns of each quadrant gives the general sense of patterning.

## Going From Hypothetical Topology to Testable Equations in Two Dimensions

The lines drawn in Figure 10 with our negative affect example are much more complex null clines (polynomial or other nonlinear forms). By reversing the null cline process, we can identify the implied equations. For example, Figure 10 implies a very complex equation form (*x* to the 3rd power for each null cline equation). Once each equation form is identified, placing all the terms on one side of the equation with 0 on the other (the exact reversal of the null cline procedure) identifies the hypothetical equations. Control parameters can then be added as appropriately or in the identification of the null clines themselves.

The multilevel model we tested for this topology (which resulted in the topology shown in Figure 11) was Equation 10 (see below). This is a simple translation of each of the cubic functions with the addition of linear crossover effects. These crossover relationships (the $b_4$s) allow for the swirling relationship we hypothesized for the second attractor.

We just went from equation to topology. Our argument is that there is utility going from the other direction. The process is merely reversed. Figure 10 included the null clines overlaid on our theory-based topology. To draw these, we knew that the lines had

$$\Delta FemaleNA_{it} = b_{f0} + b_{f1}FNA_{it} + b_{f2}FNA_{it}^2 + b_{f3}FNA_{it}^3 + b_{f4}MFA_{it} + e_{fit},$$
$$\Delta MaleNA_{it} = b_{m0} + b_{m1}MNA_{it} + b_{m2}MNA_{it}^2 + b_{m3}MNA_{it}^3 + b_{m4}FFA_{it} + e_{mit}. \quad (10)$$

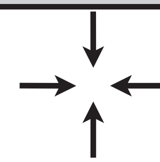
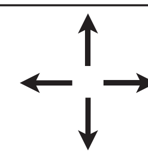
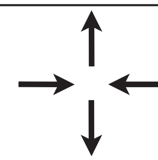
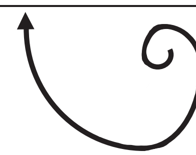
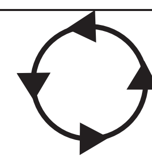
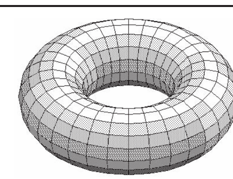
| Type | First Lyapunov (+/-) | Second Lyapunov (+/-) | Equation Components | Illustration |
|---|---|---|---|---|
| Attractor | - | - | two negative first order equations |  |
| Repellor | + | + | two positive first order equations |  |
| Saddle | - | + | one negative and one positive first order equations |  |
| Spiral Attractor | - | i | one negative first order equation and one second order equation |  |
| Spiral Repellor | + | i | one positive first order equation and one second order equation |  |
| Limit Cycle | i | 0 | one second order equation |  |
| Torus | i | i | two second order equations |  |

*Figure 19.* Combinations of Lyapunov exponents and two-dimensional typologies.



to cross at each of the set points—each of the attractors and the saddle between them. This forced two curved lines with cubic forms. Next, swirled patterns as proposed in the high–high attractor only occur when the values of one person's affect were allowed to predict their partner's affect. Together this implied two cubic equations, with the ability of each partner's negative affect also to predict that of his or her partner. This was the equation form we tested that produced the topology in Figure 11. As with the heavy drinking example, we can identify where the set points are along with the Lyapunovs in each direction, and the resultant figure (which is restricted to the data range) implies an update to our theory.

## Beyond Two Dimensions

While it is possible to go beyond two dimensions in creating topology, going beyond three is difficult to represent graphically. Beyond two dimensions new topological features become plausible (e.g., chaotic attractors become viable at three dimensions). However, many common quantitative practices are analogous to reducing the number of dimensions needed. For example, event sampling (Reis & Gable, 2000) is a sampling method where measurements are taken when a certain event occurs. This is akin to a physicist's stroboscope where other variables are examined when one variable is at its peak (or some other signal point). The result is to separate out the outcome relating to the event from the topology—the topology depicts behavior under the event.

The process of making scale scores and differences between outcomes can also reduce the number of topological dimensions (Chow et al., 2005). For example, Kelso (1995) measured the position of the right and left index fingers of participants (actually the position of batons held in left and right hands) as they attempted to move their fingers back and forth in-phase or antiphase (e.g., like two pendulums). Instead of attempting to graph both trajectories over time, Kelso graphed the difference in the phase between the two fingers at each point in time. Each finger has its own complex dynamics, but focusing on the phasic difference of the finger positions (as a difference in their position on a cycle) simplified the topology.

We can also simplify the number of dimensions necessary by allowing the hidden dimensions of topology to represent higher orders of derivatives, differences, and autoregressive relationships. For example, LDE commonly expresses change equations where acceleration (the second derivative) is the outcome. These second-order models essentially convert flow and pattern behavior (e.g., limit cycles) to have the properties of fixed points in topology. The result is that a bivariate model (two simultaneous equations) treating second derivatives as outcomes can actually imply a four-dimensional topology akin to the torus in Figure 19.

Recall in the one-dimensional topology representation where there was only a single equation predicting change. Behaviorally, we can really only observe fixed point behavior—attractors and repellers. Generating the same equation where acceleration in *x* is the outcome and *x* is the predictor is the direct second-order analog. If we were to identify the Lyapunov exponent, it would be the coefficient on *x*. In second-order models, the coefficient on *x* has the interpretation of frequency in squared radians where the sign is negative. Within topology, the sign is negative because a limit cycle is attractive—a stable pattern of behavior. The frequency representation has the interpretation of being a Lyapunov exponent, indicating how fast you move around the path. In essence, we get the description of a two-dimensional topological representation but in a single equation form.

In terms of topology, we have converted one dimension of the system to a hidden dimension. We can observe the topological pattern by graphing the first derivative (velocity) on the *y*-axis and *x* (position) on the *x*-axis (see Figure 20). Notice that acceleration is still hidden.

It is also common to include the first derivative (velocity) as a predictor. Doing so generates a form of the second Lyapunov exponent we would have seen in a first-order model. That is, the coefficient on velocity represents damping—how the oscillations lose or gain amplitude (distance from the set point). Combining this coefficient and the coefficient on *x* (position) provides the two Lyapunov exponents of the two-dimensional system.

Importantly, they are transforms of the Lyapunovs described earlier. Generating two actual equations where change is the outcome follows the rules of vector math. Using acceleration as the outcome follows the rules of complex numbers. Both can be used for depicting the same patterns, but in drastically different ways—in the vector approach we essentially need two variables to represent changes rather than one. In essence, this is what is occurring when additional lags are added in autoregressive style models (e.g., DFA where two lags are included as opposed to just one).

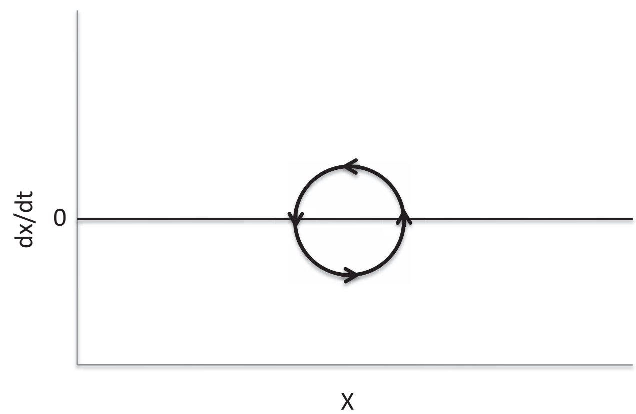



*Figure 20.* Plot of velocity in *x* on the *y*-axis and *x* on the *x*-axis. The data circles the set point, but the slope between velocity and *x* fails to depict the limit cycle.

An important concern about second-order models is that currently there is no way to extrapolate the set points of the system. The math provided for identifying the set points will instead tend to result in a value of 0. This is partly a function of current methods, since the general approach in LDE is to drop the intercept from the equations (the intercept theoretically does not exist because it would be the value of acceleration when velocity and position are both 0, which does not happen if the data is centered around the homeostatic point and cycling—when position is 0, velocity is its maximum). One possible concern from this is that current methods may lack a means of identifying if the observed behavior is a function of different topological regions or collapsing across them. One solution may be to conduct a single first-order equation first, save out residuals, and then conduct second-order modeling on the residuals with a control parameter indicating which basin of attraction a given time point is within. This method is untested and relies on a two-step procedure that may not be as accurate as an approach that would be able to estimate both first-order and second-order behavior simultaneously.

## Discussion

This article argued for converting our theories into topology, which can then be converted into testable equation form. To do so, we have described multiple features of topologies and provided a number of examples for how to transition theories or hypothesized effects into a topological representation. In addition, we outlined several tools for translating topologies into equation form, including identifying set points, the strength of the set points in terms of Lyapunov exponents, and null clines in two dimensions and outlining the role of main effects and moderators in these equations.

An ongoing question raised by using the methodology argued for in this article is, how does one choose the topology that more accurately models their phenomenon? As a theorist-seeking description of a phenomenon, the model generation approach begins by theorizing what the patterns look like in time. We suggest a process not unlike what was done in our hypothetical negative affect example. Envision the patterns of each variable through time. Identify attractors, control parameters, order parameters, and where topological features are located based on your theory. Once one has some sense of the topologies, one has to ask if they are on the same map or if the map is changing as a function of control parameters (and if control parameters are changing the map, what parts of the map are they changing)? One can then add in the topological features we tend not to see, but are implied, such as the repellers and saddles that form the edges of each basin of attraction.

Once we have a hypothetical state space of the theory, this can be converted into equation form. There are several general principles:

- The number of dimensions of the state space stipulates the number of simultaneous change equations.
- Swirls and cycles in topology require coupling relationships between equations (where x predicts changes in *y* and *y* predicts changes in *x*) or must be simplified via a methodological or quantitative technique (i.e., event sampling, phase variables, scale score or equivalent latent variables, or using higher orders of derivatives as outcomes).
- Control parameters added as main effects merely move the set points in linear models.
- Control parameters added as moderators in linear models can alter the topological feature (e.g., from attractor to repeller or merely a weaker attractor).
- Higher orders of polynomials directly correspond to more topological features (a quadratic implies two features, while a cubic implies three).
- One-dimensional topological features alternate. So a map of two attractors without a control parameter must imply a repeller between them.

Once we determine the equation form that represents the topology, this equation form can be tested on data directly. The resultant equations can then be used to generate the state space it implied—a direct reflection of the original theory.

## Estimating Derivatives

The approach outlined in this article often requires derivatives as the outcomes with longitudinal data. There are multiple methods that can be used to estimate derivatives. First, structural equation modeling (SEM) programs can be creatively used to capture derivative functions (e.g., Chow et al. (2005) constructed latent variable derivatives from multiple indicators of affect simultaneously). Measurement-wise, SEM is preferable because it can capture measurement structure that cannot be represented in virtually any other domain. However, many of the nonlinear forms outlined have yet to be explored in SEM. In SEM there are recent methods for capturing two-way interactions and polynomials (Marsh, Wen, & Hau, 2006). The extent to which these methods can be expanded to higher order forms akin to those promoted here remains an unknown.

Another method involves using approximation methods for generating derivatives and using regression and multilevel modeling approaches for model testing. The local linear approximation technique (also the generalized local linear approximation technique) promoted in the past (see Boker, 2001) works well for first-order models but becomes less desirable with the inclusion of second-order forms. Instead, we currently prefer the generalized orthogonal local derivatives approach for generating derivatives, given that it makes orthogonal estimates (see Deboeck, 2010). As illustrated here, simple differences can also work, though there is a history of debate as to the benefits and disadvantages of their statistical properties.

Finally, traditional approximation approaches have suggested detrending your data first. Detrending is a data processing technique that separates long-term changes (i.e., trends) from short-term changes in data. For example, within the oscillatory models, such as those depicted by second-order equations, it is common to model the residuals from a moving average model or linear growth model. We believe this may be less necessary to the extent that one has properly specified the first-order and second-order forms correctly. Detrending will tend to normalize all the patterns in your data, removing what may be important stable state differences, and ultimately muddying the waters as opposed to clearing them up. Instead, we believe proper centering and scaling, as it has been expanded in regression and multilevel modeling, may be better tools.

## Limitations

Our example from Littlefield et al. (2012) highlights some important considerations for working with topology in conjunction with statistical modeling. Statistical models inherently incorporate error terms to capture error in model, estimation, sampling, and measurement. Yet, our translation of equations into maps ignored this error. For example, two of Littlefield's coefficients (the crossover or coupling relationships) were nonsignificant, and yet we utilized the sample estimated values instead of a value of 0. Dynamical systems theory does maintain notions of error, but distinguishes them slightly differently by incorporating the idea of perturbations (Stewart, 2002). Perturbations are constant nudges through time due to parts of the system that remain unexamined. They are inherent in the maps in that a fixed-point attractor and a fixed-point repeller at the set point can only be distinguished under perturbations—you stay at the attractor and leave the repeller once perturbed. In essence the map guides behavior, but does not stop someone from climbing a mountain—it simply captures how much effort it would take to make that trek. Since these models incorporate stability through Lyapunov exponents, the notions of perturbations are incorporated, while other notions of error are not. The impact of capturing some, but not all, of error therefore remains an unknown.

Furthermore, we treated Littlefield's model (and our affect examples) as unbiased estimates of coefficients. The authors are unaware of any research investigating the limitations of various models in their ability to properly capture all the various topological circumstances, especially when Lyapunov exponents are $i$ (square root of $-1$; see Figure 19). The extent to which one must seek alternative procedures, such as moving to second-order models of change, will require further investigation. However, it is important to remember that maps are believed to exist on manifolds—stretchable fields (Stewart, 2002). For example, in terms of manifolds a circle and square are the same. As an analogy, there are over a dozen ways to create a map of the globe (including a three-dimensional globe). Each appears slightly different, but all are showing the same information. The same may be true for maps estimated through various statistical models. That is, some bias in one estimation procedure or another may be more a function of how we display the map rather than depict different behavior. Such notions require further investigation to understand how bias in estimation impacts the description of behaviors we observe in the maps generated from equations.

Our treatment of Littlefield et al. (2012) also regarded the relationship of change to be constant over time. In fact, Littlefield's other models freed up this temporal invariance. Underlying dynamical systems models is the assumption that an equation or set of equations can be used to characterize the changes in the system through time. These equations are more tractable when they are assumed to change under predictable ways that can be incorporated into the equations (e.g., control parameters) rather than having different equations at different instances. We therefore stuck to their first model, which met this logic.

Furthermore, we treated the time points from Littlefield et al. (2012) as equidistant, when in fact they were not. The timing of measurement is of paramount importance for systems models, such as the maps shown here. The impact is best illustrated through the stroboscope logic of event sampling mentioned earlier. Timing can turn chaos into cycles and cycles into fixed points. It is reasonable to assume that timing can also have adverse effects. For example, in time delay reconstruction (a descriptive method for extracting phase space), timing of measurement relative to the timing in which the data pattern unfolds can greatly distort the resultant maps (Kantz & Schreiber, 1997). However, it is not as simple as merely requiring timing that is evenly spaced. In fact, there are multiple timings that might apply to psychological phenomena (McGrath & Kelly, 1986). These philosophical differences may differentiate when calculus applies (the use of derivatives) versus discrete time (the use of differences). Such choices could potentially simplify the description of complex patterns through time or make them more complex.

## Conclusion

The larger aim of this article has been to show how visual tools from topology can capture a theory while also having direct mathematical translation. This allows for the integration of complex modeling techniques currently available to the theories they represent. Topology provides a way to understand these seemingly intractable modeling techniques. As a tool for the behavioral sciences, topology can ultimately provide researchers with a way to work with complex ideas without limitations and then use the topology to generate plausible complex equation forms that represent the cutting edge of how we statistically test our theories. Ideally, theory leads to model testing and model testing leads to theory adjustment. As a midstep between complex theory and models, topology may make this process much more efficient.

When Aiken and West (1991) illustrated the process of simple slopes testing, they empowered researchers with ways to go from regression interactions to graphical representations and then ways to test and explore specific locales within the scatterplot. Topology essentially provides an analog. It is a graphical representation of a series of complex relationships—substantially more complex than a regression interaction. It highlights the importance of the set points and their characteristic roots. And it provides a way to directly translate complex theory to diagram to testable equation and back again.

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# Appendix

## Glossary of Terms

**Antiphase:** Opposite phase. An example would be the two sides of a rope in a pulley. As one side moves up, the other side moves down.

**Attractiveness:** The degree to which a system or variable changes toward an attractor in time; see *Lyapunov exponent*.

**Attractor:** A state that a system or variable changes toward in time.

**Change score:** Numerical value of the difference between states; difference score. This is the simplest representation of a discrete derivative.

**Characteristic root:** Strength of a topological feature; see *Lyapunov exponent*.

**Control parameter:** Variables that have the capacity to alter the topology of a state space. An example would be the temperature of a burner on a stove. At a low temperature, you might let your hand go near the burner, but at a high temperature, you would be less likely to hold your hand near the burner. Control parameters are often thought of as the independent variables of a dynamical system.

**Cusp catastrophe:** A mathematical model in which a system can sometimes show smooth changing behavior and other times show category-like stable states. These states tend to be resistant to change, while the smooth behavior tends to function more fluidly. An example would be how attitudes change frequently when they are of low relevance to an individual, but when highly relevant they can lock in at high or low values and are resistant to change.

**Damping:** Decrease in oscillations of a system. Friction is damping for a moving object.

**Detrending:** Data processing technique that separates long-term changes from short-term changes. This usually involves calculating a moving average or general pattern of change like a growth model and only retaining the residuals from this model as the data of analysis.

**Dynamical systems:** The study of dynamics. Multicomponent systems that interact to form emergent complex patterns of change over time. An example is weather. Many factors such as temperature, humidity, and air pressure may interact to form a storm.

**Dynamics:** Change over time.

**Event sampling:** A sampling method where measurements are taken when a certain event occurs. An example would be filling out a brief series of questions whenever a person has a social interaction longer than 1 min.

**First-order model:** An equation that includes first-order derivatives (e.g., velocity or a difference), but not higher derivatives. Much of this article uses velocity as the outcome generating first-order models.

**Fixed point attractors:** The state that a system changes to be closer to a set value. If a system state is at a fixed point attractor, it will be stable. An example is a pen lying on the ground. A small bump in any direction will most likely result in a pen lying on the ground.

**Fixed point repellers:** The state that a system changes to be farther away from a set value. A system is unstable at the location of a fixed point repeller. It is extremely rare to directly observe a fixed point repeller. An example would be a pen balanced on its tip—a small change in any direction would make it fall.

**Flow:** Change of a system. An example would be the movement of a drop of water flowing down a hill.

**Homeostatic:** The tendency to maintain stability or return to a set point after perturbation. An example would be the activity of a thermostat. A thermostat activates heating and cooling systems to maintain a specified temperature.

**Homogeneity of model:** A model is unchanging in its description across people; see *Stationarity*.

**In-phase:** Completely in sync. An example would be the wheels of car when moving straight. Both wheels turn together at the same time and same rate.

**Interindividual variability:** Variability between two or more individuals. An example would be personality. People with different personalities would behave differently. There would be a typical range of personality among a population.

**Intraindividual variability:** Variability within one individual. An example would be mood. An individual's behavior would vary depending on his or her mood. An individual would have a typical range of mood.

**Latent variables:** Variables that were not measured usually captured through structural equation modeling; see also *Unobserved variables*.

(*Appendix continues*)

**Lag:** The period between one event and another. In dynamical systems, the lag between two time points is often used to model the change of a system over time depending on the earlier state. An autoregressive relationship is one form of lag relationship. Also known as a time delay approach.

**Limit cycles:** A topological feature of a state space in which a system changes in a constant repetition. An example would be the seasons.

**Lyapunov exponent:** The numerical value of the strength of a topological feature. The rate that a system will change toward or away from a particular state. In geographical terms, this would be the steepness of a slope that a marble is rolling down. Lyapunov exponents can be calculated locally (e.g., at a set point) or globally for the entire system. Global Lyapunov exponents are useful for identifying certain complex behaviors (e.g., deterministic chaos).

**Nonlinear:** In mathematics, an equation in which the terms are not of the first degree/order. The notion of nonlinear has generated some confusion in that a first-order model predicting change can generate a nonlinear pattern of change through time. We specifically use nonlinear to depict the predictor side of the equation. By adding polynomial forms and interactions, the change equations become able to depict more than one stable pattern with the same equation.

**Null cline:** Area where only one outcome is changing at a time. In terms of equations, the line in which change is fixed to 0 while ignoring the other equations in the model. There are as many null clines and dimensions to the topology.

**Oscillation:** Repetition over time.

**Path diagrams:** A diagram specifying the influence of different parameters on other parameters of a system, often used in structural equation modeling.

**Perturbation:** Small changes in a system due to parts of the system that are necessary but not modeled. They distinguish the stability of a set point in that an attractor is resistant to perturbations, while a repeller is not.

**Phase portrait:** A topographical representation of a state space or phase space where the altitude dimension represents the length of vectors. It is calculated by taking the integral of the equations that generate the state space.

**Phase space:** See *State space*.

**Repeller:** An unstable state that a system or variable moves away from in time.

**Repulsiveness:** The degree to which a system moves away from a state; see *Lyapunov exponent*.

**Saddle:** Topological feature of a state space in which a system is attractive in one direction and repulsive in the other. An example would be a triangle-shaped roof. Rain would drip in one direction on one side of the ridge and drip in the other direction on the other side of the ridge.

**Second-order model:** An equation that includes second-order derivatives (acceleration or differences of differences). Some methods treat acceleration as the outcome to capture oscillatory relationships.

**Separatrice:** See *Saddle*.

**Set point:** A topological feature upon which changes in a system can be depicted relative to that point. The set point can be defined in Cartesian coordinates of the variables that are changing in time. An example would be the temperature setting of a thermostat. When the temperature rises or falls, the thermostat activates systems to return the temperature to the set point (in this case the set point is an attractor).

**Spiral attractor:** A topological feature that combines a fixed point attractor with a limit cycle so that the state of the system spirals toward a set point.

**Spiral repeller:** A topological feature that combines a fixed point attractor with a limit cycle so that the state of the system spirals away from a set point.

**Stable state:** A state from which a system is not likely to change. An example would be a marble in the bottom of a champagne flute. The marble is not likely to move away from its current location. These could be attractors (or example), but they can also be patterns, such as the limit cycle.

**State space:** A visual graph of arrows that show where values change over time given where they start. Unlike a time series, time is integrated into the figure rather than being explicit. State spaces can be hypothetical (what our theory translates into in terms of our expectations of change), observed (plotting an arrow of each observed change), or implied by equations.

**Stationarity:** A statistical model that is unchanging in its description of a phenomenon through time; see *Homogeneity of model*.

**Strength (of topological feature):** The degree to which a system changes toward or away from an area of the state space; see also *Lyapunov exponent*.

**Torus:** Three-dimensional ring/donut shape. Two second-order equations may specify this shape as a topological feature in a state space.

**Topological feature:** A specific pattern of change within the state space of a system.

**Topological map:** Graphical representation of a set of equations.

**Topology:** The mathematics for linking maps in the form of state spaces and phase portraits to equations of change. All the equations generated are forms of calculus that can be expressed with different orders of derivatives as the equation forms.

**Trajectory:** Direction toward which a state is changing.

**Unobserved variables:** Variables that were not measured; see also *Latent variables*.

**Vector field:** See *State space*.

**Velocity flow field:** See *State space*.

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